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Fixed points of the derivative and k-th power of solutions of complex linear differential equations in the unit disc

Guowei Zhang and Ang Chen*

Abstract

In this paper we consider the question of the existence of fixed points of the derivatives of solutions of complex linear differential equations in the unit disc. This work improves some very recent results of T.-B. Cao.

Keywords: fixed points, solutions, complex linear differential equations, unit disc.
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1 Introduction and main results

In this paper, we assume that the reader is familiar with the fundamental results and the standard notations of the Nevanlinna's theory on the complex plane and in the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ (see [6, 5, 15]). Many authors have investigated the growth and oscillation of the solutions of complex linear differential equations in $\mathbb{C}$. In the unit disc, there already exist many results [11, 7, 8], but the study is more difficult than that in the complex plane, because the efficient tool, Wiman-Valiron theory, in the complex plane doesn't hold in the unit disc.

Many important results have been obtained on the fixed points of general transcendental meromorphic functions for almost four decades, see [4]. However, there are few studies on the fixed points of solutions of differential equations, specially in the unit disc. In [3], Z.-X. Chen firstly studied the problem on the fixed points and hyper-order of solutions of second order linear differential equations with entire coefficients. After that, there were some results which improve those of Z.-X. Chen, see [10, 13, 14, 9]. Recently, T.-B. Cao [1] firstly investigated the fixed points of solutions of linear complex differential equations in the unit disc. In the present paper, we continue to study the problem in the unit disc. In order to

*Department of Mathematics, Shandong University, Jinan 250100, P. R. China

E-mail address: zhirobo@yahoo.com.cn (G. W. Zhang); ang.chen.jr@gmail.com (A. Chen)

The second author is the corresponding author.

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be read more clearly, we give some definitions as following.

For $n \in \mathbb{N}$, the iterated n-order of a meromorphic function f in D is defined by

$$\sigma_n(f) = \limsup_{r \to 1^-} \frac{\log_n^+ T(r,f)}{-\log(1-r)},$$

where $\log_1^+ = \log^+ = \max\{\log x, 0\}$, $\log_{n+1}^+ x = \log^+ \log_n^+ x$. If f is analytic in D, then the iterated n-order is defined by

$$\sigma_{M,n}(f) = \limsup_{r \to 1^-} \frac{\log_{n+1}^+ M(r,f)}{-\log(1-r)}.$$

If f is analytic in D, it is well known that $\sigma_{M,1}(f)$ and $\sigma_1(f)$ satisfy the inequalities $\sigma_1(f) \leq \sigma_{M,1}(f) \leq \sigma_1(f) + 1$ which are the best possible in the sense, see [12]. However, it follows by Proposition 2.2.2 in [2] that $\sigma_{M,n}(f) = \sigma_n(f)$ for $n \geq 2$.

For $n \in \mathbb{N}$ and $a \in \mathbb{C} \bigcup \{\infty\}$, the iterated n-convergence exponent of the sequence of a-points in D of a meromorphic function f in D is defined by

$$\lambda_n(f-a) = \limsup_{r \to 1^-} \frac{\log_n^+ N(r, \frac{1}{f-a})}{-\log(1-r)}.$$

Similarly, $\overline{\lambda}_n(f-a)$, the iterated n-convergence exponent of the sequence of distinct a-points in D of a meromorphic function f in D is defined by

$$\overline{\lambda}_n(f-a) = \limsup_{r \to 1^-} \frac{\log_n^+ \overline{N}(r, \frac{1}{f-a})}{-\log(1-r)}.$$

For $n \in \mathbb{N}$, the iterated n-convergence exponent of the sequence of fixed points in D of a meromorphic function f in D is defined by

$$\tau_n(f) = \limsup_{r \to 1^-} \frac{\log_n^+ N(r, \frac{1}{f-z})}{-\log(1-r)}.$$

Similarly, $\overline{\lambda}_n(f-z)$, the iterated n-convergence exponent of the sequence of distinct fixed points in D of a meromorphic function f in D is defined by

$$\overline{\tau}_n(f) = \limsup_{r \to 1^-} \frac{\log_n^+ \overline{N}(r, \frac{1}{f-z})}{-\log(1-r)}.$$

Finally, we give the definition about the degree of small growth order of functions in D as polynomials on the complex plane. Let f be a meromorphic function in D and

$$D(f) = \limsup_{r \to 1^-} \frac{T(r,f)}{-\log(1-r)} = b.$$

If $b < \infty$, we say that f is non-admissible; if $b = \infty$, we say that f is admissible. Moreover, for $F \subset [0,1)$, the upper and lower densities of F are defined by

$$\overline{dens}_D F = \limsup_{r \to 1^-} \frac{m(F \cap [0,r))}{m([0,r))}, \ \underline{dens}_D F = \liminf_{r \to 1^-} \frac{m(F \cap [0,r))}{m([0,r))},$$

respectively, where $m(G) = \int_G \frac{dt}{1-t}$ for $G \subset [0, 1)$.

In [1], T.-B. Cao investigated the fast growth of the solutions of high order complex differential linear equation with analytic coefficients of n-iterated order in the unit disc. For using the results of T.-B. Cao conveniently, we write them in the following form. T.-B. Cao considered the equation

$$f^{(k)} + A(z)f = 0 \tag{1.1}$$

where $A(z)$ is analytic function in D, and proved the following theorems;

Theorem A. [1] *Let H be a set of complex numbers satisfying $\overline{dens}_D\{|z| : z \in H \subseteq D\} > 0$, and let $A(z)$ be an analytic function in D such that $\sigma_{M,n}(A) = \sigma < \infty$ and for constant α we have, for all $\varepsilon > 0$ sufficiently small,*

$$|A(z)| \geq \exp_n\{\alpha(\frac{1}{1-|z|})^{\sigma-\varepsilon}\}$$

as $|z| \to 1^-$ for $z \in H$. Then every nontrivial solution f of (1.1) satisfies $\sigma_n(f) = \infty$ and $\sigma_{n+1}(f) = \sigma$.

Theorem B. [1] *Let H be a set of complex numbers satisfying $\overline{dens}_D\{|z| : z \in H \subseteq D\} > 0$, and let $A(z)$ be an analytic function in D such that $\sigma_n(A) = \sigma < \infty$ and for constant α we have, for all $\varepsilon > 0$ sufficiently small,*

$$T(r, A(z)) \geq \exp_{n-1}\{\alpha(\frac{1}{1-|z|})^{\sigma-\varepsilon}\}$$

as $|z| \to 1^-$ for $z \in H$. Then every nontrivial solution f of (1.1) satisfies $\sigma_n(f) = \infty$ and $\sigma_{M,n}(A) \geq \sigma_{n+1}(f) \geq \sigma$.

In [1], T.-B. Cao also investigated the fixed points of the solutions of high order complex differential linear equation with analytic coefficients in the unit disc and proposed that: How about the fixed points and iterated order of differential polynomials generated by solutions of linear differential equations in the unit disc. In the present paper we consider the derivatives of the solutions of the equations and get some theorems as following:

Theorem 1.1. *Let the assumptions of Theorem A hold and assume also that f is a nontrivial solution of Equation (1.1). Then*

$$\tau_n(f^{(i)}) = \overline{\tau}_n(f^{(i)}) = \lambda_n(f^{(i)} - z) = \overline{\lambda}_n(f^{(i)} - z) = \sigma_n(f) = \infty, \tag{1.2}$$

$$\begin{aligned} \tau_{n+1}(f^{(i)}) &= \overline{\tau}_{n+1}(f^{(i)}) = \lambda_{n+1}(f^{(i)} - z) = \overline{\lambda}_{n+1}(f^{(i)} - z) \\ &= \sigma_{n+1}(f) = \sigma. \end{aligned} \tag{1.3}$$

Theorem 1.2. *Let the assumptions of Theorem B hold and assume also that f is a nontrivial solution of Equation (1.1). Then*

$$\tau_n(f^{(i)}) = \overline{\tau}_n(f^{(i)}) = \lambda_n(f^{(i)} - z) = \overline{\lambda}_n(f^{(i)} - z) = \sigma_n(f) = \infty, \tag{1.4}$$

$$
\begin{aligned}
\sigma_{M,n}(A) \geq \tau_{n+1}(f^{(i)}) &= \overline{\tau}_{n+1}(f^{(i)}) = \lambda_{n+1}(f^{(i)} - z) \\
&= \overline{\lambda}_{n+1}(f^{(i)} - z) = \sigma_{n+1}(f) \geq \sigma. \qquad (1.5)
\end{aligned}
$$

In addition, we study the fixed points of f^k, here f is a nontrivial solution of equation

$$f'' + A(z)f = 0, \qquad (1.6)$$

where $A(z)$ is an analytic function in D. We get our theorems as following:

Theorem 1.3. *Let the assumptions of Theorem A hold and assume also that f is a nontrivial solution of Equation (1.6). Then*

$$\tau_n(f^k) = \overline{\tau}_n(f^k) = \lambda_n(f^k - z) = \overline{\lambda}_n(f^k - z) = \sigma_n(f) = \infty, \qquad (1.7)$$

$$\tau_{n+1}(f^k) = \overline{\tau}_{n+1}(f^k) = \lambda_{n+1}(f^k - z) = \overline{\lambda}_{n+1}(f^k - z) = \sigma_{n+1}(f) = \sigma. \qquad (1.8)$$

Theorem 1.4. *Let the assumptions of Theorem B hold and assume also that f is a nontrivial solution of Equation (1.6). Then*

$$\tau_n(f^k) = \overline{\tau}_n(f^k) = \lambda_n(f^k - z) = \overline{\lambda}_n(f^k - z) = \sigma_n(f) = \infty, \qquad (1.9)$$

$$
\begin{aligned}
\sigma_{M,n}(A) \geq \tau_{n+1}(f^k) &= \overline{\tau}_{n+1}(f^k) = \lambda_{n+1}(f^k - z) = \overline{\lambda}_{n+1}(f^k - z) \\
&= \sigma_{n+1}(f) \geq \sigma. \qquad (1.10)
\end{aligned}
$$

2 Preliminary lemmas

Lemma 2.1. [6] *Let f be a meromorphic function in the unit disc, and let $k \in \mathbb{N}$. Then*

$$m(r, \frac{f^{(k)}}{f}) = S(r, f),$$

where $S(r, f) = O(\log^+ T(r, f)) + O(\log(\frac{1}{1-r}))$, possibly outside a set $E \subset [0, 1)$ with $\int_E \frac{dr}{1-r} < \infty$. If f is of finite order (namely, finite iterated 1-order) of growth, then

$$m(r, \frac{f^{(k)}}{f}) = O(\log(\frac{1}{1-r})).$$

Lemma 2.2. [10] *Suppose that $f(z)$ is a nonzero solution of equation (1.1) and $k \geq 2$. Let $\omega_i = f^{(i)} - z,\ i = 0, 1, \cdots, k-2$. Then w_i satisfy the following equations*

$$\sum_{j=0}^{i} H_{ij}(A)\omega_i^{(k-j)} - A^{i+1}\omega_i = zA^{i+1}, \qquad (2.1)$$

where $H_{ij}(A)$ $(j=0,1,\cdots,i)$ *are differential polynomials of* A.

Lemma 2.3. [10] *Suppose that* $f(z)$ *is a nonzero solution of equation (1.1) and* $k \geq 2$. *Let* $\omega_{k-1} = f^{(k-1)} - z$. *Then* w_{k-1} *satisfies the following equation*

$$\sum_{j=0}^{k-1} H_{(k-1)j}(A)\omega_{k-1}^{(k-j)} - A^k\omega_{k-1} = zA^k - H_{(k-1)(k-1)}(A), \tag{2.2}$$

where $H_{(k-1)j}(A)$ $(j=0,1,\cdots,k-1)$ *are differential polynomials of* A.

Lemma 2.4. [3] *Suppose that* $f(z)$ *is a nonzero solution of equation (1.1) and* $k \geq 2$. *Let* $\omega_k = f^{(k)} - z$. *Then* w_k *satisfies the following equation*

$$\begin{aligned}\sum_{j=0}^{k-1} H_{kj}(A)\omega_k^{(k-j)}(H_{kk}(A) - A^{k+1})\omega_k &= zA^{k+1} \\ &\quad -H_{k(k-1)}(A) - zH_{kk}(A),\end{aligned} \tag{2.3}$$

where $H_{kj}(A)$ $(j=0,1,\cdots,k-1)$ *and* $H_{kk}(A)$ *are differential polynomials of* A.

Lemma 2.5. [13] *Suppose that* $f(z)$ *is a nonzero solution of equation (1.6) and* $k \geq 2$. *Let* $g = f^k - z$. *Then* g *satisfies the following equation*

$$g''g + zg'' - \frac{k-1}{k}(g')^2 - 2\frac{k-1}{k}(g') + kAg^2 + 2kAzg = \frac{k-1}{k} - kAz^2.$$

3 Proof of Theorems

Proof of Theorem 1.1 and 1.2

Proof. We will prove Theorem 1.1 and 1.2 together. Suppose that $f(z)$ is a nontrivial solution of equation (1.1). Set $\omega_i = f^{(i)} - z,\ i = 0,\ 1,\ \cdots,\ k$, then for every i, a point z_0 is a fixed points of $f^{(i)}$ if and only if z_0 is a zero of ω_i, and

$$\sigma_n(\omega_i) = \sigma_n(f^{(i)}) = \sigma_n(f),\ \overline{\tau}_n(f^{(i)}) = \overline{\lambda}_n(\omega_i),$$

$$\sigma_{n_1}(\omega_i) = \sigma_{n+1}(f^{(i)}) = \sigma_{n+1}(f),\ \overline{\tau}_{n+1}(f^{(i)}) = \overline{\lambda}_{n+1}(\omega_i),$$

By Lemma 2.2, Lemma 2.3 and Lemma 2.4, we know that $\omega_i,\ i = 0,\ 1,\ \cdots,\ k$ satisfy the following equations :

$$\sum_{j=0}^{i} H_{ij}(A)\omega_i^{(k-j)} - A^{i+1}\omega_i = zA^{i+1},\ (i = 0,1,\cdots,k-2). \tag{3.1}$$

$$\sum_{j=0}^{k-1} H_{(k-1)j}(A)\omega_{k-1}^{(k-j)} - A^k\omega_{k-1} = zA^k - H_{(k-1)(k-1)}(A). \tag{3.2}$$

$$\sum_{j=0}^{k-1} H_{kj}(A)\omega_k^{(k-j)}(H_{kk}(A)-A^{k+1})\omega_k = zA^{k+1} -H_{k(k-1)}(A)-zH_{kk}(A). \tag{3.3}$$

where $H_{ij}(A)$ are differential polynomials of A. Since A is admissible analytic functions in D, we have $zA^{i+1}\not\equiv 0,\ i=0,\ 1,\ \cdots,\ k-2$. We claim that

$$zA^k - H_{(k-1)(k-1)}(A)\not\equiv 0,$$

and

$$zA^{k+1}-H_{k(k-1)}(A)-zH_{kk}(A)\not\equiv 0.$$

In fact, if $zA^k - H_{(k-1)(k-1)}(A)\equiv 0$, rewrite it as

$$A=\frac{H_{(k-1)(k-1)}(A)}{zA^{k-1}}.$$

Since A is analytic in D, we have $T(r,A)=m(r,A)=S(r,A)$ in D, which is impossible. If $zA^{k+1}-H_{k(k-1)}(A)-zH_{kk}(A)\not\equiv 0$, rewrite it as

$$A=\frac{H_{k(k-1)}(A)}{zA^k}+\frac{H_{kk}(A)}{A^k}.$$

Similarly, we obtain that $T(r,A)=m(r,A)=S(r,A)$ in D. Hence our claim holds.

Rewrite the above equations (3.1)-(3.3) as

$$\frac{1}{\omega_i}=\frac{1}{zA^{i+1}}(\sum_{j=0}^{i}H_{ij}(A)\frac{\omega_i^{(k-j)}}{\omega_i}-A^{i+1}),\ (i=0,1,\cdots,k-2). \tag{3.4}$$

$$\frac{1}{\omega_{k-1}}=\frac{1}{zA^k-H_{(k-1)(k-1)}(A)}(\sum_{j=0}^{k-1}H_{(k-1)j}(A)\frac{\omega_{k-1}^{(k-j)}}{\omega_{k-1}}-A^k), \tag{3.5}$$

$$\frac{1}{\omega_k} = \frac{1}{zA^{k+1}-H_{k(k-1)}(A)-zH_{kk}(A)}\cdot (\sum_{j=0}^{k-1}H_{kj}(A)\frac{\omega_k^{(k-j)}}{\omega_k}+H_{kk}(A)-A^{k+1}). \tag{3.6}$$

By Lemma 2.1, there exists $E\subset[0,1)$ with $\int_E \frac{dr}{1-r}<\infty$, such that for $r\notin E$, we have,

$$m(r,\frac{1}{\omega_i})\le O(m(r,\frac{1}{A}))+C(\log^+ T(r,\omega_i)+\log\frac{1}{1-r}), \tag{3.7}$$

here $i=0,1,\cdots,k$. By the assumption, we know that A and $H_{ij}(A)$ are analytic in D. Now we split three cases to discuss the zeros of ω_i.

Case 1: for $i=0,1,\cdots,k-2$, if ω_i has a zero at $z_0\in D$ of order $m(>k)$, then from (3.4) we know z_0 is a zero of zA^{i+1} of order at least $m-k$. Hence we have

$$N(r,\frac{1}{\omega_i})\le k\overline{N}(r,\frac{1}{\omega_i})+N(r,\frac{1}{zA^{i+1}}). \tag{3.8}$$

Case 2: for $i = k-1$, if ω_i has a zero at $z_0 \in D$ of order $m(>k)$, then from (3.5) we see z_0 is a zero of $zA^k - H_{(k-1)(k-1)}(A)$ of order at least $m-k$. Hence we have

$$N(r, \frac{1}{\omega_i}) \leq k\overline{N}(r, \frac{1}{\omega_i}) + N(r, \frac{1}{zA^k - H_{(k-1)(k-1)}(A)}). \tag{3.9}$$

Case 3: for $i = k$, if ω_i has a zero at $z_0 \in D$ of order $m(>k)$, then we get from (3.6) that $zA^{k+1} - H_{k(k-1)}(A) - zH_{kk}(A)$ has zeros at z_0 of order at least $m-k$. Hence we have

$$N(r, \frac{1}{\omega_i}) \leq k\overline{N}(r, \frac{1}{\omega_i}) + N(r, \frac{1}{zA^{k+1} - H_{k(k-1)}(A) - zH_{kk}(A)}). \tag{3.10}$$

Note that for $r \to 1^-$, $C(\log^+ T(r, \omega_i) + \log \frac{1}{1-r}) \leq \frac{1}{2}T(r, \omega_i)$, where C is a constant. Thus we have

$$T(r, \omega_i) \leq 2k\overline{N}(r, \frac{1}{\omega_i}) + O(T(r, A)), \ (i = 0, 1, \cdots, k.) \tag{3.11}$$

Under the hypotheses of Theorem 1.1, we know that $\sigma_n(f) = \infty$ by Theorem A. Then, for any given sufficiently large positive number $N > \sigma + 1$, there exists $\{r'_n\}(r'_n \to 1^-)$ such that

$$\sigma_n(f) = \limsup_{r'_n \to 1^-} \frac{\log_n^+ T(r'_n, f)}{-\log(1-r'_n)} \geq N.$$

Set $\int_E \frac{dr}{1-r} = \log \delta < \infty$. Since $\int_{r'_n}^{1-\frac{1-r'_n}{\delta+1}} \frac{dr}{1-r} = \log(\delta+1)$, then there exists $r_n \in [r'_n, 1 - \frac{1-r'_n}{\delta+1}] \setminus E \subset [0, 1)$, such that

$$\frac{\log_n^+ T(r_n, f)}{-\log(1-r_n)} \geq \frac{\log_n^+ T(r'_n, f)}{\log \frac{\delta+1}{1-r'_n}} = \frac{\log_n^+ T(r'_n, f)}{\log \frac{1}{1-r'_n} + \log(\delta+1)}.$$

Hence, we have

$$\liminf_{r'_n \to 1^-} \frac{\log_n^+ T(r_n, f)}{-\log(1-r_n)} \geq \limsup_{r'_n \to 1^-} \frac{\log_n^+ T(r'_n, f)}{\log \frac{1}{1-r'_n} + \log(\delta+1)} \geq N.$$

It yields

$$\lim_{r'_n \to 1^-} \frac{\log_n^+ T(r_n, f)}{-\log(1-r_n)} \geq N.$$

Since $\sigma_n(f) \leq \sigma_{M,n}(f) = \sigma$, for any given ε, we have $T(r_n, A) \leq \exp_n(\frac{1}{1-r_n})^{\sigma+\varepsilon}$. So, for $r_n \in [r'_n, 1 - \frac{1-r'_n}{\delta+1}] \setminus E \subset [0, 1)$, we get

$$\frac{T(r_n, A)}{T(r_n, f)} \leq \frac{\exp_n(\frac{1}{1-r_n})^{\sigma+\varepsilon}}{\exp_n(\frac{1}{1-r_n})^N} \to 0.$$

So we have $T(r, A) = o(T(r, f))$. Since $\sigma_n(\omega_i) = \sigma_n(f)$, we have $T(r, A) = o(T(r, \omega_i))$. From (3.11), we have

$$\overline{\lambda}_n(\omega_i) = \lambda_n(\omega_i) = \sigma_n(\omega_i), \tag{3.12}$$

and

$$\overline{\lambda}_{n+1}(\omega_i) = \lambda_{n+1}\omega_i) = \sigma_{n+1}(\omega_i). \tag{3.13}$$

Combining (3.12), (3.13) with Theorem A, we can get (1.2) and (1.3). Thus we complete the proof of Theorem 1.1. Combining (3.12), (3.13) with Theorem B, we can get (1.4) and (1.5), which are the results of Theorem 1.2. □

Proof of Theorem 1.3 and 1.4

Proof. Suppose that $f(z)$ is a nonzero solution of equation (1.6) and $k \geq 2$. Let $g = f^k - z$. By Lemma 2.5, g satisfies the following equation

$$\begin{aligned} g''g + zg'' - \frac{k-1}{k}(g')^2 - 2\frac{k-1}{k}(g') + kAg^2 + 2kAzg &= \frac{k-1}{k} - \\ & \quad kAz^2. \end{aligned} \tag{3.14}$$

Obviously, we know that $\frac{k-1}{k} - kAz^2 \not\equiv 0$. For z satisfying $|g(z)| < 1$, we have $\frac{1}{|g(z)|} > 1$. From (3.14), we have

$$\begin{aligned} \frac{\frac{k-1}{k} - kAz^2}{|g|} \leq (1+|z|)\frac{|g''|}{|g|} &+ \frac{k-1}{k}(\frac{|g'|}{|g|})^2 + 2\frac{k-1}{k}\frac{|g'|}{|g|} \\ &+ k|A| + 2k|Az|. \end{aligned} \tag{3.15}$$

From (3.15) we have

$$m(r, \frac{1}{g}) \leq O(\sum_{i=1}^{2} m(r, \frac{g^{(i)}}{g})) + O(m(r, A)). \tag{3.16}$$

From (3.14), we know that if z_0 is a zero of g with multiplicity m, then z_0 is a zero of $\frac{k-1}{k} - kAz^2$ with multiplicity at least $m-2$. So we have

$$N(r, \frac{1}{g}) \leq 2\overline{N}(r, \frac{1}{g}) + N(r, \frac{1}{\frac{k-1}{k} - kAz^2}). \tag{3.17}$$

From (3.16) and (3.17), we have

$$T(r, g) \leq 2\overline{N}(r, \frac{1}{g}) + O(T(r, A)) + S(r, g). \tag{3.18}$$

From (3.18), we have

$$\sigma_n(g) = \lambda_n(g) = \overline{\lambda}_n(g) = \tau_n(f^k) = \overline{\tau}_n(f^k). \tag{3.19}$$

and

$$\sigma_{n+1}(g) = \lambda_n(g) = \overline{\lambda}_{n+1}(g) = \tau_n(f^k) = \overline{\tau}_{n+1}(f^k). \tag{3.20}$$

Since

$$\sigma_n(g) = \sigma_n(f^k) = \sigma_n(f) \tag{3.21}$$

From (3.19)- (3.21), and Theorem A (Theorem B), we can get Theorem 1.3 (Theorem 1.4). □

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