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Oscillation of the Solutions of Neutral Impulsive Differential-Difference Equations of First Order

M.B.Dimitrova

Technical University, 8800 Sliven, Bulgaria

D.Mishev

*Department of Mathematics, Prairie View A & M University,
Prairie View, TX 77446, USA*

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Abstract

Sufficient conditions for oscillation of all solutions of a class of neutral impulsive differential-difference equations of first order with deviating argument and fixed moments of impulse effect are found .

1. Introduction

The impulsive differential equations describe processes which are characterized as continuous, as jump-wise change of the phase variables describing the process. They are adequate mathematical models of processes and phenomena studied in theoretical physics, chemical technology, population dynamics, technique and economics. That is why, the impulsive differential equations are an object of intensive investigation.

In the recent two decades the number of investigations of the oscillatory and non-oscillatory behavior of the solutions of functional differential equations is constantly growing. Greater part of the works on this subject published by 1977 are given in [19]. In the monographs [18] and [17] published in 1987 and 1991 respectively, the oscillatory and asymptotic properties of the solutions of various classes of functional differential equations are systematically studied.

The investigation of impulsive differential equations of neutral type is still not well studied. Let us note that in contrast to [9] the present paper deals with the oscillatory properties of more general homogeneous impulsive differential equation. In the works [1]—[6],[8],[16],[17] more general necessary and sufficient conditions for oscillation and non-oscillation of the solutions of impulsive differential equations of first and second order are found.

While qualitative theory for retarded and advanced differential equations has been well developed over the last twenty years, (see,for example, [17],[18] and [19]), only in resent years has much effort been devoted to the study of neutral differential equations (see, for example, [7] and [10]-[16]).

In the present paper, we establish sufficient conditions for oscillation of all solutions of a class of neutral impulsive differential-difference equations of first order with deviating argument and fixed moments of impulse effect.

2. Preliminary notes

Consider the impulsive differential-difference inequalities of neutral type with a constant delay:

$$\begin{aligned} &[y(t)+p(t)y(t-\tau)]'+q(t)y(t-\sigma)<0, && t\neq\tau_k, k\in N,\\ &\Delta[y(\tau_k)+p_k y(\tau_k-\tau)]+q_k y(\tau_k-\sigma)<0, && k\in N, \end{aligned} \tag{1}$$

and

$$\begin{aligned} &[y(t)+p(t)y(t-\tau)]'+q(t)y(t-\sigma)>0, && t\neq\tau_k, k\in N,\\ &\Delta[y(\tau_k)+p_k y(\tau_k-\tau)]+q_k y(\tau_k-\sigma)>0, && k\in N, \end{aligned} \tag{2}$$

and corresponding to it equation

$$\begin{aligned} &[y(t)+p(t)y(t-\tau)]'+q(t)y(t-\sigma)=0, && t\neq\tau_k, k\in N,\\ &\Delta[y(\tau_k)+p_k y(\tau_k-\tau)]+q_k y(\tau_k-\sigma)=0, && k\in N, \end{aligned} \tag{3}$$

where τ, $\sigma\in R_+, R_+=(0,+\infty); \tau>\sigma$; $\tau_1,\tau_2,\ldots\tau_k,\ldots$ are the moments of impulse effect; p_k and q_k are constants ($k\in N$).

Here
$\Delta[y(\tau_k)+p_k y(\tau_k-\tau)] = y(\tau_k+0)+p_k y(\tau_k-\tau+0)-y(\tau_k-0)-p_k y(\tau_k-\tau-0)$. We suppose that $y(\tau_k-0)=y(\tau_k)$ and $y(\tau_k-\tau-0)=y(\tau_k-\tau)$ for $k\in N$.

We denote by $PC(R_+,R)$ the set of all functions $u\colon R_+\to R$, which are continuous for $t\in R_+$, $t\neq\tau_k$, $k\in N$, continuous from the left-side for $t\in R_+$ and have discontinuity of the first kind at the points $\tau_k\in R_+$, $k\in N$.

Introduce the following conditions:

H1. $0<\tau_1<\tau_2<\ldots$ and $\lim_{k\to+\infty}\tau_k=+\infty$.

H2. $p \in PC(R_+, R)$ and $p_k = p(\tau_k - 0) = p(\tau_k)$ for $k \in N$.

H3. $q \in C(R_+, R_+)$ and $q_k \geq 0$ for $k \in N$.

H4. $p(t) \leq -1$ for $t \in R_+$.

H5.

$$\int_0^{+\infty} q(s)ds + \sum_{k=1}^{+\infty} q_k = +\infty.$$

Definition 1 A function $y : [-\tau, +\infty) \to R$ is said to be a solution of (3) with initial function $\varphi \in C([-\tau, 0], R)$ if $y(t) = \phi(t)$ for $t \in [-\tau, 0]$, $y \in PC(R_+, R)$, $z(t) = y(t) + p(t)y(t-\tau)$ is continuously differentiable for $t \in R_+$, and $y(t)$ satisfies (3) for all sufficiently large $t \geq 0$.

Definition 2 The nonzero solution $y(t)$ of the equation (3) is said to be *nonoscillating* if there exists a point $t_0 \geq 0$ such that $y(t)$ has a constant sign for $t \geq t_0$. Otherwise the solution $y(t)$ is said to *oscillate.*

Definition 3 The solution $y(t)$ of the equation (3) is said to be *regular,* if it is defined on some interval $[T_y, +\infty) \subset [t_0, +\infty)$ and

$$\sup \{|y(t)|: \ t \geq T\} > 0 \quad \text{for each } T \geq T_y.$$

Definition 4 The regular solution $y(t)$ of the equation (3) is said to be *eventually positive(eventually negative)*, if there exists $t_1 > 0$ such that $y(t) > 0$ $(y(t) < 0)$ for $t \geq t_1$.

3. Main results

Theorem 1 *Let the following conditions hold:*

1. Conditions H1 — H5 *are met.*

2.

$$\liminf_{t \to +\infty} [\sum_{t \leq \tau_k < t+\tau-\sigma} (1 - \frac{q_k}{p_{k_1}}) \int_t^{t+\tau-\sigma} \frac{-q(s)}{p(s+\tau-\sigma)} ds] > \frac{1}{e}$$

where $e = exp$, $\quad p_{k_1} = p(\tau_k + \tau - \sigma)$.

Then:

1. The inequality (1) has no eventually positive solution.

2. The inequality (2) has no eventually negative solution.

3.All solutions of the equation (3) are oscillatory.

Proof. First of all,we shall prove that the inequality (1) has no eventually positive solution. Let us suppose the opposite, i.e., there exists a solution $y(t)$ of inequality (1) and a number $t_0 > 0$ such that $y(t)$ is defined for $t \geq t_0$ and $y(t) > 0$ for $t \geq t_0$.

Set

$$
\begin{aligned}
&z(t) = y(t) + p(t)y(t-\tau), \qquad \text{t} \geq 0 \\
&\Delta z(\tau_k) = \Delta[y(\tau_k) + p_k y(\tau_k - \tau)], \qquad k \in N.
\end{aligned} \tag{4}
$$

From (1) and conditions H2 and H3 it follows that

$$z'(t) < -q(t)y(t-\sigma) < 0$$

and

$$\Delta z(\tau_k) < -q_k y(\tau_k - \sigma) \leq 0.$$

The last inequalities implies that z is a decreasing function for $t \geq t_0$.

Let us suppose that $z(t) \geq 0$ for $t \geq t_1 \geq t_0$. From (4) and condition H4 we obtain

$$y(t) \geq -p(t)y(t-\tau) \geq y(t-\tau)$$

$$\Delta y(\tau_k) \geq -p_k \Delta y(\tau_k - \tau) \geq \Delta y(\tau_k - \tau),$$

i.e. y is a bounded function from below by $m > 0$.

Integrating (1) from t_1 to t $(t \geq t_1)$, we obtain

$$z(t) - z(t_1) - \sum_{t_1 \leq \tau_k < t} \Delta z(\tau_k) + \int_{t_1}^{t} q(s)y(s-\sigma)ds < 0$$

or

$$z(t) - z(t_1) + \sum_{t_1 \leq \tau_k < t} q_k y(\tau_k - \sigma) + \int_{t_1}^{t} q(s)y(s-\sigma)ds < 0,$$

i.e.,

$$z(t) \leq z(t_1) - m[\int\limits_{t_1}^{t} q(s)ds + \sum_{t_1 \leq \tau_k < t} q_k].$$

It follows from the above inequality after passing to limit as $t \to +\infty$ that $\lim_{t\to+\infty} z(t) = -\infty$, which contradicts the assumption that $z(t) \geq 0$ for $t \geq t_1$. Therefore, $z(t) < 0$, for $t \geq t_1$.

From (4) we have that $z(t) > p(t)y(t-\tau), t \geq t_1$, i.e. $z(t+\tau-\sigma) > p(t+\tau-\sigma)y(t-\sigma)$.

Multiplying both sides of the last inequality by $\dfrac{q(t)}{p(t+\tau-\sigma)} < 0$ we obtain

$$\frac{q(t)}{p(t+\tau-\sigma)} z(t+\tau-\sigma)) < q(t)y(t-\sigma) < -z'(t)$$

Therefore

$$z'(t) + \frac{q(t)}{p(t+\tau-\sigma)} z(t+\tau-\sigma)) < 0, \qquad t \geq t_1, t \neq \tau_k \tag{5}$$

But $z(\tau_k + \tau - \sigma) > p_{k_1} y(\tau_k - \sigma), \qquad p_{k_1} = p(\tau_k + \tau - \sigma)$
or

$$\frac{q_k}{p_{k_1}} z(\tau_k + \tau - \sigma) < q_k y(\tau_k - \sigma) < -\Delta z(\tau_k),$$

i.e.,

$$\Delta z(\tau_k) + \frac{q_k}{p_{k_1}} z(\tau_k + \tau - \sigma) < 0, \qquad k \in N. \tag{6}$$

Denote $\tau - \sigma = l$, $s(t) = \dfrac{-q(t)}{p(t+l)} > 0$ for $t \geq t_1$ and $s_k = \dfrac{-q_k}{p_{k_1}} > 0$, $k \in N$. Then from (5) and (6) follows that

$$\begin{aligned} &z'(t) - s(t)z(t+l) < 0, \qquad t \geq t_1, t \neq \tau_k \\ &\Delta z(\tau_k) - s_k z(\tau_k + l) < 0, \qquad \tau_k \geq t_1, \qquad k \in N. \end{aligned} \tag{7}$$

We shall prove that the impulsive differential-difference inequality (7) has no eventually negative solution. Let us suppose the opposite, i.e., there exists a solution $z(t)$ of inequality (7) and a number $t_2 > 0$ such that $z(t)$ is defined for $t \geq t_2$ and $z(t) < 0$ for $t \geq t_2$.

We divide (7) by $z(t) < 0, (t \geq t_2)$ and obtain

$$\frac{z'(t)}{z(t)} - s(t)\frac{z(t+l)}{z(t)} > 0, \qquad t \geq t_2, t \neq \tau_k \tag{8}$$

Denote

$$w(t) = \frac{z(t+l)}{z(t)}, \qquad t \geq t_2$$

From the fact that $z(t)$ is a decreasing function for $t \geq t_2$ it follows the inequality $w(t) > 1$ for $t \geq t_2$.

Integrating (8) from t to $t+l$, $(t \geq t_2)$ we obtain

$$ln\frac{z(t+l)}{z(t)} + \sum_{t \leq \tau_k < t+l} ln\frac{z(\tau_k)}{z(\tau_k+0)} > \int_t^{t+l} s(u)\frac{z(u+l)}{z(u)} du. \tag{9}$$

From

$$\Delta z(\tau_k) = z(\tau_k + 0) - z(\tau_k) < s_k z(\tau_k + l) < s_k z(\tau_k)$$

implies

$$z(\tau_k + 0) < (1 + s_k) z(\tau_k).$$

Then

$$ln\frac{z(\tau_k)}{z(\tau_k+0)} < ln\frac{1}{1+s_k}, \qquad k \in N. \tag{10}$$

From (9) and (10) we obtain

$$\int\limits_t^{t+l} \frac{z(u+l)}{z(u)} s(u)du < ln[\frac{z(t+l)}{z(t)} \prod_{t_1 \le \tau_k < t+l} \frac{1}{1+s_k}]$$

or

$$\frac{z(t+2l)}{z(t)} \int\limits_t^{t+l} s(u)du < ln[\frac{z(t+l)}{z(t)} \prod_{t \le \tau_k < t+l} \frac{1}{1+s_k}] \tag{11}$$

Using the inequality $e^x > ex$ for $x > 0$ and (11) we find that

$$\frac{z(t+l)}{z(t)\prod_{t_1 \le \tau_k < t+l}(1+s_k)} > e^{\frac{z(t+2l)}{z(t)} \int\limits_t^{t+l} s(u)du} > e\frac{z(t+2l)}{z(t)} \int\limits_t^{t+l} s(u)du$$

or

$$\frac{1}{e} > [\prod_{t \le \tau_k < t+l} (1+s_k)] \frac{z(t+2l)}{z(t+l)} \int\limits_t^{t+l} s(u)du$$

i.e.

$$\frac{1}{e} > \prod_{t \le \tau_k < t+l} (1+s_k) \int\limits_t^{t+l} s(u)du$$

The last inequality contradicts condition 2 of Theorem 1. Thus $z(t) < 0$ will not hold for all $t \ge t_2$, and therefore (1) has no eventually positive solution.

In order to prove that (2) has no eventually negative solution, it is enough to note that if $y(t)$ is a solution of (2), then $-y(t)$ is a solution of (1).

It follows from assertions 1 and 2 of Theorem 1 that the equation (3) has neither eventually positive nor eventually negative solution. Therefore, each regular solution of (3) is oscillatory.

The proof of the theorem is complete.

Corollary 1 *Let the following conditions hold:*

1. *Conditions* H1 — H5 *are met.*
2. *There exists a constant* $r > 0$ *such that*

$$\frac{q(t)}{p(t+\tau-\sigma)} \le -r$$

for $t \in R_+$.

3.

$$\liminf_{t\to\infty}\Big[\prod_{t\le\tau_k<t+\tau-\sigma}(1-\frac{q_k}{p_{k_1}})\Big] > \frac{1}{er(\tau-\sigma)}$$

where $p_{k_1} = p(\tau_k+\tau-\sigma)$.

Then:

1. The inequality (1) has no eventually positive solution.

2. The inequality (2) has no eventually negative solution.

3.All solutions of the equation (3) are oscillatory.

Proof. Let $y(t)$ be an eventually positive solution of the inequality (1) for $t \ge t_0, (t_0 > 0)$. Then, proceeding as in proof of Theorem 1, we conclude (7). From (7) and from condition 2 of Corollary 1 we obtain

$$\begin{aligned} &z'(t) - rz(t+l) < 0 \qquad t \ge t_1, t \ne \tau_k \\ &\Delta z(\tau_k) - r_k z(\tau_k+l) < 0, \qquad \tau_k \ge t_1 \qquad k \in N, \end{aligned} \tag{12}$$

where

$$r_k = -\frac{q_k}{p_{k_1}}, \qquad k \in N.$$

We shall prove that the impulsive differential-difference inequality (12) has no eventually negative solution. Let us suppose the opposite, i.e., there exists a solution $z(t)$ of inequality (12) and a number $t_2 > t_1$ such that $z(t) < 0$ for $t \ge t_2$. Then it follows from (12) that $z(t)$ is decreasing function for $t \ge t_2$.

We divide (12) by $z(t) < 0$, integrate from t to $t+l, (t \ge t_2)$ and obtain

$$ln\frac{z(t+l)}{z(t)} - \sum_{t\le\tau_k<t+l} ln\frac{z(\tau_k+0)}{z(\tau_k)} > r\int_t^{t+l}\frac{z(u+l)}{z(u)}du \tag{13}$$

Analogously to the proof of Theorem 1 we obtain that

$$r\int_t^{t+l}\frac{z(u+l)}{z(u)}du < ln\Big[\frac{z(t+l)}{z(t)}\prod_{t\le\tau_k<t+l}(1+r_k)^{-1}\Big]$$

or

$$rl\frac{z(t+2l)}{z(t)} < ln\Big[\frac{z(t+l)}{z(t)}\prod_{t\le\tau_k<t+l}(1+r_k)^{-1}\Big] \tag{14}$$

Using the monotonicity of the function $z(t)$ for $t \ge t_2$ we find that

$\dfrac{z(t+2l)}{z(t)} > 1$ and $\dfrac{z(t+l)}{z(t)} > 1$. From two last inequalities, (14) and from the inequality $e^x > ex$ for $x > 0$ we find that

$$\frac{z(t+l)}{z(t)} \prod_{t \le \tau_k < t+l} (1+r_k)^{-1} > e^{rl\frac{z(t+2l)}{z(t)}} > erl\frac{z(t+2l)}{z(t)};$$

$$\prod_{t \le \tau_k < t+l} (1+r_k)^{-1} > erl\frac{z(t)z(t+2l)}{z(t+l)z(t)},$$

or

$$\prod_{t \le \tau_k < t+l} (1+r_k)^{-1} > erl\frac{z(t+2l)}{z(t+l)}.$$

Using the monotonicity of the function $z(t)$ for $t \ge t_2$ we find that $\dfrac{z(t+2l)}{z(t+l)} > 1$.

Therefore

$$\prod_{t \le \tau_k < t+l} (1+r_k)^{-1} > erl$$

or

$$\prod_{t \le \tau_k < t+l} (1+r_k) < \frac{1}{rle}$$

The last inequality contradicts condition 3 of Corollary 1.

In order to prove that (2) has no eventually negative solution, it is enough to note that if $y(t)$ is a solution of (2), then $-y(t)$ is a solution of (1).

It follows from assertions 1 and 2 of Corollary 1 that the equation (3) has neither eventually positive nor eventually negative solution. Therefore, each regular solution of (3) is oscillatory.

The proof of the corollary is complete.

Theorem 2 *Let the following conditions hold:*

1. *Conditions* H1 — H5 *are met.*
2. $$\limsup_{k\to+\infty} [\int\limits_{\tau_k-\tau+\sigma}^{\tau_k} \frac{-q(s)}{p(s+\tau-\sigma)}ds + \sum_{\tau_k-\tau+\sigma\le\tau_i<\tau_k} (-\frac{q_i}{p_i})] > 1.$$

Then:

1. *The inequality (1) has no eventually positive solution.*
2. *The inequality (2) has no eventually negative solution.*
3. *All solutions of the equation (3) are oscillatory.*

Proof. Let $y(t)$ be an eventually positive solution of the inequality (1) for $t \ge t_0, (t_0 > 0)$. Then, proceeding as in proof of Theorem 1, we conclude (7).

We shall prove that the impulsive differential-difference inequality (7) has no eventually negative solution.

Let us suppose the opposite, i.e., there exists a solution $z(t)$ of inequality (7) and a number $t_2 > t_1$ such that $z(t) < 0$ for $t \geq t_2, t_2 \geq t_1$.

Integrating (7) from $\tau_k - l$ to τ_k ($\tau_k \geq t_2 + l, l = \tau - \sigma$), we obtain

$$\int\limits_{\tau_k - l}^{\tau_k} s(u)z(u+l)du - z(\tau_k) + z(\tau_k - l) + \sum_{\tau_k - l \leq \tau_i < \tau_k} \Delta z(\tau_i) > 0$$

or

$$z(\tau_k)\int\limits_{\tau_k - l}^{\tau_k} s(u)du - z(\tau_k) + z(\tau_k - l) + \sum_{\tau_k - l \leq \tau_i < \tau_k} s_i z(\tau_i + l) > 0$$

i.e.,

$$z(\tau_k)[\int\limits_{\tau_k - l}^{\tau_k} s(u)du + \sum_{\tau_k - l \leq \tau_i < \tau_k} s_i - 1] > -z(\tau_k - l) > 0$$

From the last inequality and from $z(t) < 0$ we obtain

$$\int\limits_{\tau_k - l}^{\tau_k} s(u)du + \sum_{\tau_k - l \leq \tau_i < \tau_k} s_i < 1$$

The last inequality contradicts condition 2 of Theorem 2.

In order to prove that (2) has no eventually negative solution, it is enough to note that if $y(t)$ is a solution of (2), then $-y(t)$ is a solutionof (1).

It follows from assertions 1 and 2 of Theorem 2 that the equation (3) has neither eventually positive nor eventually negative solution. Therefore, each regular solution of (3) is oscillatory.

The proof of the theorem is complete.

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References

[1] Bainov D.D., Dimitrova M.B., *Sufficient conditions for oscillation of the solutions of a class of impulsive differential equations with deviating argument,* J. of Appl. Math. and Stochastic Analysis, Vol.9, No.1, Spring 1996, 33 -42.

[2] Bainov D.D., Dimitrova M.B., *Oscillation of sub- and superlinear impulsive differential equations with constant delay,* Applicable Analysis, Vol. 64, 1997, 57 - 67.

[3] Bainov D.D., Dimitrova M.B., *Oscillation of the solutions of impulsive differential equations and inequalities with a retarded argument,* Rocy Mountain J. of Math.,Vol. 28,No. 1, 1998, 25 - 40.

[4] Bainov D.D., Dimitrova M.B., *Oscillatory properties of the solutions of impulsive differential equations with a deviating argument and Nonconstant Coefficients,* Rocy Mountain J. of Math., Vol. 27, No. 4, 1997, 1027 - 1040.

[5] Bainov D.D., Dimitrova M.B.,Dishliev A., *Necessary and sufficient conditions for existence of nonoscillatory solutions of a class of impulsive differential equations of second order with retarded argument,* Applicable Analysis, Vol. 63, 1996, 287 -297.

[6] Bainov D.D., Dimitrova M.B.,Dishliev A., *Asymptotic properties of the solutions of a class of impulsive differential equations of second order with retarded argument,* Kodai Math. J.,Vol. 20, No. 2, 1997, 120 -126.

[7] Bainov D.D., Mishev D.P., *Oscillation theory for neutral differential equations with delay,* Adam Hilger, Bristol, 1991.

[8] Berezansky L., Braverman E., Oscillation of a linear delay impulsive differential equations, Communications on Appl. Nonl. Anal., 3(1996),61-77.

[9] Gopalsamy K., Zhang B. G., *On delay differential equations with impulses,* J. Math. Anal. Appl., 139 (1989), 110–122.

[10] Graef J.R., Grammatikopoulos M.K., Spikes P.W., *Behavior of the nonoscillatory solutions of first order neutral delay differential equations ,* Differential Equations, Proceeding of the EQUADIFF Conference (1989), 265 - 272, Dekker, New York.

[11] Graef J.R., Grammatikopoulos M.K., Spikes P.W., *On the behavior of a first order nonlinear neutral delay differential equations ,* Applicable Anal. 40 (1991), 111 - 121.

[12] Graef J.R., Grammatikopoulos M.K., Spikes P.W., *Asymptotic and oscillatory behavior of first order nonlinear neutral delay differential equations ,* J. Math. Anal. Appl., 155,(1991), 562 - 571.

[13] Grammatikopoulos M.K., Grove E.A., Ladas G., *Oscillations of a first order nonlinear neutral delay differential equations ,*J. Math. Anal. Appl., 120,(1986), 510 - 520.

[14] Grammatikopoulos M.K., Grove E.A., Ladas G., *Oscillation and asymptotic behavior of neutral differential equations with deviating arguments,* Appl. Anal. 22 (1986), 1 - 19.

[15] Grammatikopoulos M.K., Rashkov P.I., *First order nonlinear neutral differential inequalities with oscillating coefficients,* Adv. Math. Sci. Appl., Gakkotosho, Tokio, Vol. 6, No. 2 (1996),653 - 688.

[16] Grammatikopoulos M.K., Stavroulakis I.P., *Necessary and sufficient conditions for oscillation of neutral equations with deviating arguments,* J. London Math. Soc. 41 (1990), 244 - 260.

[17] I. Györi, G. Ladas, *Oscillation theory of delay differential equations with applications,* Clarendon Press, Oxford, 1991.

[18] G. S. Ladde, V. Lakshmikantham, B. G. Zhang, *Oscillation theory of differential equations with deviating arguments,* Pure and Applied Mathematics, Vol. **110,** Marcel Dekker, 1987.

[19] V. N. Shevelo, *Oscillations of solutions of differential equations with deviating arguments,* Naukova Dumka, Kiev, 1978 (in Russian).

(Received September 8, 2004)

Email addresses:

dimichev@pvamu.edu

MBDimitrova@tu-sliven.com