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Corrections to "On Lévy processes conditioned to stay positive"

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Abstract

We correct two errors of omission in our paper, [2] .

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We would like to correct two errors of omission in our paper, [2]. The first occurs in equation (2.4), where we overlooked the possibility that the downgoing ladder time process has a positive drift. This happens if and only if 0 is not regular for $(0,\infty)$. If this drift is denoted by η, the correct version of (2.4) is

$$\begin{aligned} &\mathbb{P}_x(\tau_{(-\infty,0)} > \mathbf{e}/\varepsilon) = \mathbb{P}(\underline{X}_{\mathbf{e}/\varepsilon} \geq -x) \\ &= \mathbb{E}\left(\int_{[0,\infty)} e^{-\epsilon t} \mathbb{1}_{\{\underline{X}_t \geq -x\}}\, d\underline{L}_t\right) [\eta\varepsilon + \underline{n}(\mathbf{e}/\varepsilon < \zeta)]\,, \end{aligned} \tag{1}$$

and the correct version of (2.5) is:

$$h(x) = \lim_{\varepsilon\to 0} \frac{\mathbb{P}_x(\tau_{(-\infty,0)} > \mathbf{e}/\varepsilon)}{\eta\varepsilon + \underline{n}(\mathbf{e}/\varepsilon < \zeta)}\,. \tag{2}$$

However this makes no essential difference to the proof of the following Lemma 1: we just need to replace $\underline{n}(\mathbf{e}_\varepsilon < \zeta)$ by $\eta\varepsilon + \underline{n}(\mathbf{e}_\varepsilon < \zeta)$ four times, and $\underline{n}(\zeta)$ by $\eta + \underline{n}(\zeta)$ in (2.6). The details can be seen in section 8.2 of [3]. We should also mention that (1) can be found in [1]: see equation (8), p 174.

The second omission is that we failed to give any proof of

Corollary 1. *Assume that 0 is regular upwards. For any $t > 0$ and for any $\mathcal{F}_t$ -measurable, continuous and bounded functional F,*

$$\underline{n}(F, t < \zeta) = k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}F).$$

The clear implication from our paper is that this follows immediately from our main result, Theorem 2, but this overlooks the singularity at zero of the function $1/h(x)$. Since this Corollary has been cited in a number of recent papers, we give here a full proof of it.

Proof. From (3.2) and Theorem 2 of [2] we see that, for any fixed $\delta > 0, t > 0$,

$$\underline{n}(F, t < \zeta, X_t > \delta) = k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}F, X_t > \delta),$$

and in particular, taking $F \equiv 1$,

$$\begin{aligned} \underline{n}(t < \zeta, X_t > \delta) &= k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}, X_t > \delta) \\ &= k \lim_{x\to 0} \mathbb{P}_x(X_t > \delta, \tau_{(-\infty,0)} > t)/h(x). \end{aligned}$$

Suppose we can show that

$$\underline{n}(t < \zeta) = k \lim_{x\to 0} \mathbb{P}_x(\tau_{(-\infty,0)} > t)/h(x). \tag{3}$$

Then, by subtraction,

$$\begin{aligned} \underline{n}(t < \zeta, X_t \leq \delta) &= k \lim_{x\to 0} \mathbb{P}_x(X_t \leq \delta, \tau_{(-\infty,0)} > t)/h(x) \\ &= k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}, X_t \leq \delta). \end{aligned}$$

Since $\underline{n}(t < \zeta, X_t = 0) = 0$, if K is an upper bound for F, we also have

$$\lim_{\delta \to 0} \underline{n}(F, t < \zeta, X_t \le \delta) \le K \lim_{\delta \to 0} \underline{n}(t < \zeta, X_t \le \delta) = 0,$$

and the required conclusion follows.

To prove (3) we start with (1), and, since we are assuming that 0 is regular upwards, the drift η in the downwards ladder time process is zero, so we can write it as

$$\int_0^\infty e^{-\varepsilon t} \mathbb{P}_x(\tau_{(-\infty,0)} > t) dt = h^{(\varepsilon)}(x) \int_0^\infty e^{-\varepsilon t} \underline{n}(\zeta > t) dt,$$

where $h^{(\varepsilon)}(x) = \mathbb{E}\left(\int_0^\infty e^{-\varepsilon t} \mathbb{I}_{\underline{X}_t \ge -x} dL_t \right)$. We know $0 \le h^{(\varepsilon)}(x) \le h(x)$, so

$$\int_0^\infty e^{-\varepsilon t} \mathbb{P}_x(\tau_{(-\infty,0)} > t) dt \le h(x) \int_0^\infty e^{-\varepsilon t} \underline{n}(\zeta > t) dt.$$

But we also have

$$\begin{aligned} \liminf_{x \downarrow 0} \frac{\mathbb{P}_x(\tau_{(-\infty,0)} > t)}{h(x)} &\ge \lim_{\delta \downarrow 0} \lim_{x \downarrow 0} \frac{\mathbb{P}_x(\tau_{(-\infty,0)} > t, X_t > \delta)}{h(x)} \\ &= \lim_{\delta \downarrow 0} \underline{n}(\zeta > t, X_t > \delta) = \underline{n}(\zeta > t). \end{aligned}$$

Together, these prove that

$$\lim_{x \downarrow 0} \int_0^\infty e^{-\varepsilon t} \frac{\mathbb{P}_x(\tau_{(-\infty,0)} > t) dt}{h(x)} = \int_0^\infty e^{-\varepsilon t} \underline{n}(\zeta > t) dt.$$

Thus the measure with density $\mathbb{P}_x(\tau_{(-\infty,0)} > t)/h(x)$ converges weakly to the measure with the continuous density $\underline{n}(\zeta > t)$. But if $0 < c < t$ are fixed we have

$$\begin{aligned} \lim_{x \to 0} \mathbb{P}_x(\tau_{(-\infty,0)} &> t)/h(x) \ge c^{-1} \lim_{x \to 0} \int_t^{t+c} \mathbb{P}_x(\tau_{(-\infty,0)} > s) ds / h(x) \\ &= c^{-1} \int_t^{t+c} \underline{n}(\zeta > s) ds \ge \underline{n}(\zeta > t + c), \\ \lim_{x \to 0} \mathbb{P}_x(\tau_{(-\infty,0)} &> t)/h(x) \le c^{-1} \lim_{x \to 0} \int_{t-c}^{t} \mathbb{P}_x(\tau_{(-\infty,0)} > s) ds / h(x) \\ &= c^{-1} \int_{t-c}^{t} \underline{n}(\zeta > s) ds \le \underline{n}(\zeta > t - c), \end{aligned}$$

and letting $c \downarrow 0$ we conclude that (3) holds, and hence the Corollary. $\square$

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References

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