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SUPPORT OF A MARCUS EQUATION IN DIMENSION 1

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Abstract

The purpose of this note is to give a support theorem in the Skorohod space for a one-dimensional Marcus differential equation driven by a Lévy process, without any assumption on the latter. We also give a criterion ensuring that the support of the equation is the whole Skorohod space. This improves, in dimension 1, a result of H. Kunita.

Compared to the immense literature on diffusion processes, the mathematical corpus dealing with stochastic differential equations of jump-type remains somewhat poor, and it is of course very tempting to see if some classical results for diffusions cannot be adapted to the discontinuous case. An example for this is Stroock-Varadhan's support theorem [15], which was partially generalized by H. Kunita [7] and the present author [12] for stochastic equations driven by Lévy processes. In addition to regularity and boundedness assumptions on the coefficients of the equation, in both articles some conditions had to be put on the driving process (more precisely, on its jumping measure) to obtain the result.

In this paper we aim to get rid of these conditions for a certain family of one-dimensional differential equations (the "Marcus equations", see below for a precise definition) driven by Lévy processes. The support theorem is proved in full generality on the latter, and under weaker assumptions on the coefficients, close to the minimal ones ensuring the existence of a unique strong solution to the equation. In particular, this improves in dimension 1 the results of [7], which dealt precisely with the Marcus equation.

An advantage of our proof is also its simplicity: thanks to a pathwise representation à la Doss-Sussmann, which was recently put into light by Errami-Russo-Vallois [3] (see also [6]), one can namely view the Marcus equation as a certain locally Lipschitz functional of the driving Lévy process, whose support in the Skorohod space was already described in [13]. The support of the equation is then obtained by transfering the support of the Lévy process through this functional, after having isolated the jumps. Unfortunately, it seems difficult to use this method for general non-linear Itô equations (see the remarks at the end of this paper), for which the statement of the support theorem is conjectured in [12].

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We now introduce our equation. Let $\{Z_t,\, t \geq 0\}$ be a real-valued Lévy process starting from 0. Recall its Lévy-Itô decomposition: for every $t \geq 0$,

$$Z_t \;=\; \alpha t \;+\; \beta W_t \;+\; \int_0^t \int_{|z|\leq 1} z\, \widetilde{\mu}(ds, dz) \;+\; \int_0^t \int_{|z|>1} z\, \mu(ds, dz),$$

where $\alpha, \beta \in \mathbb{R}$, W is a linear Brownian motion starting from 0, ν is a positive measure on $\mathbb{R} - \{0\}$ satisfying

$$\int_{\mathbb{R}} \frac{|z|^2}{|z|^2+1}\, \nu(dz) \;<\; \infty,$$

μ is the Poisson measure on $\mathbb{R}^+ \times \mathbb{R}$ with intensity $ds \otimes \nu(dz)$, and $\widetilde{\mu} = \mu - ds \otimes \nu$ is the compensated measure. In the following, $\mathbb{P}$ will denote the law of the process Z in the space $\mathcal{D}$ of càdlàg functions from $\mathbb{R}^+$ to $\mathbb{R}$ starting from 0. For every càdlàg process X and every $T \geq 0$ we will also use the standard notations:

$$|X|_T^* \;=\; \sup_{t\leq T} |X_t| \quad \text{and} \quad [X]_T^d \;=\; \sum_{t \leq T} |\Delta X_t|^2.$$

Let $a : \mathbb{R} \to \mathbb{R}$ be a locally Lipschitz function with linear growth and $b : \mathbb{R} \to \mathbb{R}$ a $\mathcal{C}^1$ function with bounded and locally Lipschitz derivative. For every $x \in \mathbb{R}$ we consider the following S.D.E. driven by Z:

$$X_t \;\;=\;\; x \;+\; \int_0^t a\,(X_s)\, ds \;+\; \int_0^t b\,(X_{s-}) \,\circ\, dZ_s. \tag{1}$$

In the above expression, the second integral must be interpreted as follows:

$$\int_0^t b\,(X_{s-}) \,\circ\, dZ_s \;=\; \int_0^t b\,(X_{s-})\, dZ_s \;+\; \frac{\beta^2}{2} \int_0^t bb'\,(X_s)\, ds$$
$$+ \sum_{0<s\leq t} \left\{ \rho\,(b\Delta Z_s, X_{s-}) - X_{s-} - b\,(X_{s-})\Delta Z_s \right\},$$

where the first term in the right-hand expression is a classical Itô integral, and where for every u global Lipschitz function and $x \in \mathbb{R}$ we noted $\rho(u,x) = f(u;x,1)$, $f(u;x,.) : \mathbb{R} \to \mathbb{R}$ being the unique solution of the following differential equation:

$$\begin{cases} f(u;x,0) \;=\; x \\[2ex] \dfrac{d}{dt}\, f(u;x,t) \;=\; u\,[f(u;x,t)] \quad \text{for every } \; t \in \mathbb{R}. \end{cases}$$

Notice that the non-compensated sum above is well-defined, since by Taylor's formula

$$\sum_{0<s\leq t} \left\{ \rho(X_{s-}, b\Delta Z_s) - X_{s-} - b\,(X_{s-})\Delta Z_s \right\} \;\leq\; \frac{1}{2} \left(\sup_{s\leq t,\, 0\leq\theta\leq 1} \left| bb'\,(f(b\Delta Z_s; X_{s-}, \theta)) \right| \right) [Z]_t^d,$$

and the right-hand expression is finite a.s. Without loss of generality on (1), we can suppose that $\alpha = 0$ in the Lévy-Itô decomposition of Z, which will be done subsequently.

Roughly, the behaviour of the process X solution of (1) is the following: when Z does not jump, X solves a classical Stratonovitch differential equation driven by βW with coefficients a for the drift and b for the diffusion. When Z jumps, X also jumps according to:

$$\Delta X_t \;=\; c(X_{t-}, \Delta Z_t)$$

where c is defined by $c(u,v) = \rho(bv, u) - u$ for every $u, v \in \mathbb{R}$.
The equation (1) was introduced by Marcus [10] [11]. It can be viewed as a generalization for càdlàg driving processes of the Stratonovitch equation, even though it only concerns a specific class of integrands. We refer to [9] for a systematic study of (1), where the driving process is a general semimartingale on $\mathbb{R}^d$. In the literature, (1) is sometimes called a "canonical" equation, since in contrast to a classical Itô-Meyer differential equation, it can be defined on manifolds in an intrinsic way [1].
Recently, Errami-Russo-Vallois [3] studied this equation in dimension 1, and in the very general framework where the driving càdlàg process has just finite quadratic variation. Under the above conditions on a and b, they gave in particular a pathwise representation of the solution, using the well-known method of Doss [2] for Stratonovitch equations driven by continuous martingales. All the computations of the fifth section of [3] are indeed similar to those of [2]. Putting together the Theorem 5.4 of [3] and the Lemme 4 of [2] yields the following:

Theorem A (Doss, Errami-Russo-Vallois) *The equation* (1) *has a unique strong solution X, whose sample paths can be represented by*

$$X \;=\; \Phi(x, Z)$$

where $\Phi : \mathbb{R} \times \mathcal{D} \to \mathcal{D}$ is a locally Lipschitz functional for the locally uniform norm.

Remarks. (a) Compensating the sum of the jumps, one can see (1) as a classical non-linear Itô equation. Under the conditions on a and b, the existence of a unique strong solution is ensured by the usual criteria [4].
(b) Obviously, for every $t > 0$, X_t depends only on $\{Z_s,\, s \le t\}$. We will write

$$X_t \;=\; \Phi_t\left(x, \{Z_s,\, s \le t\}\right).$$

(c) The local Lipschitz property of Φ means: for every $T > 0$, for every compact set of $\mathbb{R} \times \mathcal{D}([0,T], \mathbb{R})$ with respect to the norm $|.| + |.|^*_T$, there exists a constant K such that for every $(x,u), (y,v)$ in this compact set,

$$|\Phi(x,u) - \Phi(y,v)|^*_T \;\le\; K\left(|x-y| + |u-v|^*_T\right).$$

We now briefly come back to Lévy processes. We will say that Z is of type I if its sample paths have a.s. finite variations on every compact time interval, i.e. if $\beta = 0$ and if

$$\int_{\mathbb{R}} |z|\, \nu(dz) \;<\; \infty.$$

In the other case we will say that Z is of type II. The following is an easy consequence of the main result of [13]:

Theorem B *Suppose that Z is of type II. For every function $\psi : \mathbb{R}^+ \to \mathbb{R}$ continuous null at* 0, *every $T > 0$ and $\varepsilon > 0$*

$$\mathbb{P}\left[\,|Z - \psi\,|^*_T < \varepsilon\,\right] \quad > \quad 0.$$

In the following we will combine Theorems A and B to prove a support theorem for the equation (1). The idea is not new and had already been used in [2] to retrieve Stroock-Varadhan's support theorem for diffusions in dimension 1. However, the situation is here a bit more complicated since we are dealing with càdlàg processes.
As a random variable, X solution of (1) is namely valued in $\mathcal{D}_x$, the space of càdlàg functions from $\mathbb{R}^+$ to $\mathbb{R}$ starting at x. We want to define its topological support with respect to $\mathbf{d}$, the local Skorohod-Prokhorov distance: for every $f, g \in \mathcal{D}_x$

$$\mathbf{d}(f,g) \;=\; \sum_{n\geq 1} 2^{-n}(1\wedge \mathbf{d}_n(f,g))$$

where of every $n \in \mathbb{N}^*$ $\mathbf{d}_n$ is a certain functional involving the restrictions of f, g on the time set $[0, n+1]$ and increasing changes of time. We refer to [5] pp. 293-294 for a precise definition of this functional. By definition Supp X is the set of $\phi \in \mathcal{D}_x$ such that for every $n \in \mathbb{N}^*$, $\varepsilon > 0$,

$$\mathbb{P}\left[\,\mathbf{d}_n(X,\phi) < \varepsilon\,\right] \;>\; 0.$$

As in [15], we aim to characterize Supp X in terms of ordinary differential equations. We thus define $\mathbf{U}$ the set of sequences $u = \{u_p\} = \{t_p, z_p\}$, where $\{t_p\}$ is an increasing sequence of $(0, +\infty)$ tending to $+\infty$ and $\{z_p\}$ any sequence in Supp $\nu \,-\, \{0\}$. For every $u \in \mathbf{U}$ and every continuous function $\psi : \mathbb{R}^+ \to \mathbb{R}$, we introduce the following piecewise differential equation:

$$\phi_t \;=\; x \;+\; \int_0^t a\,(\phi_s)\,ds \;+\; \int_0^t b\,(\phi_s)\,\psi_s\,ds \;+\; \sum_{t_p\leq t} c\left(\phi_{t_p-}, z_p\right). \tag{2}$$

Under the conditions on a, b and by the Cauchy-Lipschitz theorem, there is a unique global solution to (2). Let $\mathcal{S}$ be the set of solutions to (2), u varying in $\mathbf{U}$ and ψ in the set of continuous functions from $\mathbb{R}^+$ to $\mathbb{R}$. Let $\overline{\mathcal{S}}$ be the closure of $\mathcal{S}$ in $(\mathcal{D}_x, \mathbf{d})$.

Theorem *Suppose Z is of type II. Then*

$$\text{Supp } X \;=\; \overline{\mathcal{S}}.$$

Remarks. (a) When Z is of type I a characterization of the support of X can be found in [12], also in terms of piecewise O.D.E.'s. Roughly one must consider the same type of differential equations as above, but without diffusion part: $\psi \equiv 0$. Actually this paper deals with general non-linear Itô equations, and in this framework it also gives a (complicated) support theorem in the case of type II, but only for $\beta = 0$ and when ν is almost symmetric or satisfies an assumption of regular variation.
(b) Under stronger assumptions on a, b and in the case $\beta \neq 0$, the above result had been obtained by H. Kunita [7] when a horizon T is fixed for the equation (1). His method consists in generalizing the approximative continuity results of [15]. The case $\beta = 0$ is also considered and yields an analogous support theorem to that of [12], but under the following additional assumption:

$$\int_{\eta_n\leq|z|\leq 1} z\,\nu(dz)$$

must have a limit for some sequence $\{\eta_n\}$ tending to 0.

Proof of the Theorem. The inclusion Supp $X \subset \overline{\mathcal{S}}$ is a routine which relies on a Wong-Zakai approximation [9]. We leave it to the reader. The reverse inclusion is *a priori* more delicate, but Theorems A and B will make things somewhat easier.
Fix $n \in \mathbb{N}^*$, $\varepsilon > 0$, $u \in \mathbf{U}$, and ψ continuous from $\mathbb{R}^+$ to $\mathbb{R}$. Let ϕ be the solution of (2) given by u and ψ. Let $N_n \in \mathbb{N}^*$ be such that

$$0 < t_1 < \ldots < t_{N_n} \leq n+1$$

are the jumping times of ϕ before $n+1$. Set

$$\eta \;=\; \inf\{|z_i|,\, i = 1, \ldots, N_n\}/2 \quad \text{and} \quad \xi_t \;=\; \int_0^t \psi_s\, ds.$$

Introduce the process $\{Z_t^\eta,\, t \geq 0\}$ defined by

$$Z_t^\eta \;=\; \int_0^t \int_{|z| \geq \eta} z\, \mu(ds, dz)$$

for every $t \geq 0$, and define $\{T_p, Z_p\}$ the sequence of its successive jumping times and values. Set $\tilde{\phi}$ for the solution of (2) where $\{t_p\}$ is replaced by $\{T_p\}$. For every $\rho > 0$, the event

$$\Omega_\rho^1 \;=\; \left\{ \sup_{1 \leq p \leq N_n + 1} |T_p - t_p| < \rho \right\}$$

has a positive $\mathbb{P}$-probability. Introducing an appropriate increasing change of time transforming each t_i into T_i for $i = 1, \ldots, N_n$, and using Gronwall's lemma repeatedly, it is not difficult to see that for $\rho > 0$ small enough,

$$\mathbf{d}_n\left(\phi, \tilde{\phi}\right) \;<\; \varepsilon/2$$

on Ω_ρ^1. We next set

$$\Omega_\rho^2 \;=\; \left\{ \sup_{1 \leq p \leq N_n} |Z_p - z_p| < \rho \right\}.$$

For every $\rho > 0$, this event has positive $\mathbb{P}$-probability as well, since the sequence $\{z_p\}$ is chosen in Supp ν. We finally introduce $\tilde{Z}^\eta = Z - Z^\eta$. It is a Lévy process of type II and $\xi : \mathbb{R}^+ \to \mathbb{R}$ is continuous null at 0. Thanks to Theorem B the event

$$\Omega_\rho^3 \;=\; \left\{ \left| \tilde{Z}^\eta - \xi \right|_{n+1}^* < \rho \right\}$$

has thus also positive $\mathbb{P}$-probability for every $\rho > 0$. We will show that there exists $\rho > 0$ such that on $\Omega_\rho = \Omega_\rho^1 \cap \Omega_\rho^2 \cap \Omega_\rho^3$,

$$\left| X - \tilde{\phi} \right|_{n+1}^* \;<\; \varepsilon/2. \tag{3}$$

Since obviously Ω_ρ^1, Ω_ρ^2 and Ω_ρ^3 are mutually independent under $\mathbb{P}$, and since the Sup distance on $[0, n+1]$ dominates $\mathbf{d}_n$, we will then have

$$\mathbb{P}\left[\, \mathbf{d}_n\,(X, \phi) < \varepsilon \,\right] \;>\; 0,$$

which is the desired result.
In order to show (3), we first remark that since ξ is $\mathcal{C}^1$, $\zeta = \Phi(x,\xi)$ solves the differential equation

$$\zeta_t \;=\; x \;+\; \int_0^t a(\zeta_s)\,ds \;+\; \int_0^t b(\zeta_s)\,\psi_s\,ds.$$

In particular, $\tilde{\phi}_t = \zeta_t = \Phi_t\left(x, \{\xi_s,\, s \le t\}\right)$ on $\{0 \le t < T_1\}$. Similarly, for every $p = 1, \ldots, N_n$, we see that on $\{0 \le t < T_{p+1} - T_p\}$,

$$\tilde{\phi}_{T_p+t} \;=\; \Phi_t\left(\tilde{\phi}_{T_p}, \left\{\xi_{T_p+s},\, s \le t\right\}\right).$$

The essential point is that an analogous decomposition yields for the process X: on $\{t < T_1\}$, $X_t = \Phi_t\left(x, \left\{\tilde{Z}^\eta_s,\, s \le t\right\}\right)$ and for every $p = 1, \ldots, N_n$, on $\{0 \le t < T_{p+1} - T_p\}$,

$$X_{T_p+t} \;=\; \Phi_t\left(X_{T_p}, \left\{\tilde{Z}^\eta_{T_p+s},\, s \le t\right\}\right).$$

Besides, it is easy to see that there exists a deterministic constant M such that for every ρ small enough,

$$\left|\tilde{Z}^\eta\right|^*_{n+1} < M$$

on Ω^3_ρ. We can now apply Theorem A, successively on $[0,T_1)$, $[T_1,T_2), \ldots,\ [T_{N_n}, n)$: there exists a deterministic constant K such that for every ρ small enough

$$\begin{cases} \left|X - \tilde{\phi}\right|^*_{T_1-} & \le\; K\left(\left|\tilde{Z}^\eta - \xi\right|^*_{T_1}\right) \\ \left|X - \tilde{\phi}\right|^*_{T_1,\,T_2-} & \le\; K\left(\left|X_{T_1} - \tilde{\phi}_{T_1}\right| + \left|\tilde{Z}^\eta - \xi\right|^*_{T_1,\,T_2}\right) \\ \vdots & \qquad \vdots \\ \left|X - \tilde{\phi}\right|^*_{T_{N_n},\,n+1} & \le\; K\left(\left|X_{T_{N_n}} - \tilde{\phi}_{T_{N_n}}\right| + \left|\tilde{Z}^\eta - \xi\right|^*_{T_{N_n},\,n+1}\right), \end{cases}$$

on Ω_ρ, where for every times $S < T$ and every process X we introduced the following notations:

$$|X|^*_{T-} \;=\; \sup_{t<T} |X_t|, \quad |X|^*_{S,\,T-} \;=\; \sup_{S \le t < T} |X_t|, \quad |X|^*_{S,\,T} \;=\; \sup_{S \le t \le T} |X_t|.$$

Recall now that for every $i = 1, \ldots, N_n$,

$$\Delta X_{T_i} \;=\; c\left(X_{T_i-}, Z_i\right) \quad \text{and} \quad \Delta\tilde{\phi}_{T_i} \;=\; c\left(\tilde{\phi}_{T_i-}, z_i\right).$$

Under the conditions on b, c is obviously a continuous function. We thus see that all the right-hand terms in the above inequalities can be made together arbitrarily small on Ω_ρ as $\rho \downarrow 0$, which completes the proof. □

We now want to give some criteria ensuring that the support of X is big enough in $\mathcal{D}_x$. We denote by $\mathcal{C}_x$ the set of continuous functions from $\mathbb{R}^+$ to $\mathbb{R}$ starting from x. The following is immediate in view of Example VI.8.1 in [4]:

Corollary 1 *Suppose that Z is of type II and that b never vanishes. Then* Supp X *is the closure of the set of following functions:*

$$\phi : t \mapsto \psi_t + \sum_{t_p \le t} c\left(\phi_{t_p-}, z_p\right),$$

where $\psi \in \mathcal{C}_x$ and $\{t_p, z_p\} \in \mathbf{U}$.

Remarks. (a) In particular, the second order integro-differential operator associated with X fulfils everywhere a strong maximum principle [15]. Analytically this is not a surprise, since this operator is "hypoelliptic" when b never vanishes. Notice also that in the case of type I the non-vanishing condition on b does not play any rôle, since there is no diffusion part in the skeleton.
(b) Under the above conditions on Z and b, it is also immediate that for every $t > 0$,

$$\text{Supp } X_t = \mathbb{R}.$$

Under additional regularity conditions on a, b and ν, one can prove that the law of X_t admits a continuous density [8]. In view of the above, one can wonder if the density will be everywhere positive.

The next corollary gives a criterion ensuring that the support of X is $\mathcal{D}_x$ itself:

Corollary 2 *Suppose Z of type II, that b never vanishes and that* Supp $\nu = \mathbb{R}$. *Then*

$$\text{Supp } X = \mathcal{D}_x.$$

Proof. Thanks to the proof of Corollary VI.1.43 a) in [5], any function in $\mathcal{D}_x$ can be approximated for the distance $\mathbf{d}$ by a sequence of step-functions with finitely many jumps on any compact time set, starting from x. In view of Corollary 1 and since Supp $\nu = \mathbb{R}$, it is thus enough to prove that for every $y \in \mathbb{R}$, the set $\{f(bz; y, 1),\ z \in \mathbb{R}\}$ is $\mathbb{R}$ itself if b never vanishes. This is an easy exercise.

□

Final remarks. (a) With a different functional, Theorem A is still valid when Z is valued in $\mathbb{R}^d$, $d \ge 1$, under the classical solvability assumption on the vector fields driving the equation [6]. If Z is "entirely of type II", i.e. if none of its projections on any subspace of $\mathbb{R}^d$ is of type I, Theorem B applies as well [13]. In this framework, it is thus also possible to obtain a support theorem for (1), with the corresponding multi-dimensional skeletons.
(b) Consider now a general non-linear Itô equation driven by Z:

$$X_t = x + \int_0^t a(X_s)\,ds + \int_0^t b(X_{s-})\,dZ_s + \sum_{0<s\le t} c(X_{s-}, \Delta Z_s) \tag{4}$$

where c is *any* continuous function satisfying locally $c(y,z) \le K|z|^2$ for some constant K. Apparently its stochastic dynamics is not so different from that of the Marcus equation, since (4) can be rewritten as follows:

$$X_t = x + \int_0^t a(X_{s-}, dV_s) + \int_0^t b(X_{s-}) \circ dZ_s$$

for some appropriate function a and some process V with finite variations. If V were continuous, Theorem 5.4 in [3] would apply, so that we could again get a Theorem A. Unfortunately this is not the case, and apart from equations involving a different class of non-linear integrands [14], the author did not succeed yet in finding a general pathwise representation for the solutions of (4).
(c) Even if a pathwise representation does not hold, one can wonder if it would not be possible to "compare" the sample paths of (1) and (4), in order to prove a support theorem for the latter. One is then rapidly led to compare (1) with the following equation:

$$X_t \;=\; x + \int_0^t a(X_{s-})\,ds + \int_0^t b(X_{s-}) \circ dZ_s + \sum_{0<s\leq t} |\Delta Z_s|^2.$$

However, even if the last term in the right-hand side has finite variations, it jumps infinitely often on any compact time interval, so that one must take in consideration the a.s. behaviour of (1) in small times. But the distance between x and X_t solution to (1) is of order $|Z_t|$ when $t \downarrow 0$, which does not converge rapidly enough to 0 if Z is of type II.

References

[1] D. Applebaum and H. Kunita. Lévy flows on manifolds and Lévy processes on Lie groups. *J. Math. Kyoto Univ.* **33**, pages 1103–1123, 1993.

[2] H. Doss. Liens entre équations différentielles stochastiques et ordinaires. *Ann. I.H.P. Prob. Stat.* **XIII**, pages 99–125, 1977.

[3] M. Errami, F. Russo and P. Vallois. Itô formula for $\mathcal{C}^{1,\lambda}$ functions of a càdlàg process and related calculus. Prépublication de l'Institut Élie Cartan, Nancy, 1999.

[4] N. Ikeda and S. Watanabe. *Stochastic differential equations and diffusion processes.* North-Holland, 1989.

[5] J. Jacod and A. N. Shiryaev. *Limit theorems for stochastic processes.* Springer, 1987.

[6] H. Kunita. Stochastic flows with self-similar properties. In Davies (ed.) et al., *Stochastic Analysis and applications. Proceedings of the 5th Grenyog symposium, July 9-14, 1995*, pages 286–300. World Scientific Publishing, Singapore, 1996.

[7] H. Kunita. Canonical stochastic differential equations based on Lévy processes and their supports. In Crauel (ed.) et al., *Stochastic dynamics. Conference on Random dynamical systems, Bremen, Germany, April 28 - May 2, 1997*, pages 283–304. Springer, New York, 1999.

[8] H. Kunita and J. P. Oh. Existence of density for canonical differential equations with jumps. Preprint, 1999.

[9] T. G. Kurtz, E. Pardoux and Ph. Protter. Stratonovich stochastic differential equations driven by general semimartingales. *Ann. I.H.P. Prob. Stat.* **31**, pages 351–377, 1995.

[10] S. I. Marcus. Modeling and analysis of stochastic differential equations driven by point processes. *I.E.E.E. Trans. Inf. Theory* **24**, pages 164–172, 1978.

[11] S. I. Marcus. Modeling and approximation of stochastic differential equations driven by semimartingales. *Stochastics* **4**, pages 223–245, 1981.

[12] Th. Simon. Support theorem for jump processes. *Stochastic Processes and their Applications* **89**, pages 1–30, 2000.

[13] Th. Simon. Sur les petites déviations d'un processus de Lévy. To appear in *Potential Analysis.*

[14] Th. Simon. Über eine nichtlineare Itô-Differentialgleichung. In preparation.

[15] D. W. Stroock and S. R. S. Varadhan. On the support of diffusion processes with applications to the strong maximum principle. In *Proc. 6th Berkeley Symp. Math. Stat. Prob.* **III**, pages 333–359. Univ. California Press, Berkeley, 1972.