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# ON POSITIVE SOLUTIONS FOR A CLASS OF STRONGLY COUPLED P-LAPLACIAN SYSTEMS

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*Dedicated to Jacqueline Fleckinger on the occasion of an international conference in her honor*

Abstract. Consider the system

$$
\begin{gathered}
-\Delta_p u = \lambda f(u,v) \quad \text{in } \Omega \\
-\Delta_q v = \lambda g(u,v) \quad \text{in } \Omega \\
u = 0 = v \quad \text{on } \partial\Omega
\end{gathered}
$$

where $\Delta_s z = \operatorname{div}(|\nabla z|^{s-2}\nabla z)$, $s > 1$, $\lambda$ is a non-negative parameter, and $\Omega$ is a bounded domain in $\mathbb{R}$ with smooth boundary $\partial\Omega$. We discuss the existence of a large positive solution for $\lambda$ large when

$$
\lim_{x\to\infty} \frac{f(x, M[g(x,x)]^{1/q-1})}{x^{p-1}} = 0
$$

for every $M > 0$, and $\lim_{x\to\infty} g(x,x)/x^{q-1} = 0$. In particular, we do not assume any sign conditions on $f(0,0)$ or $g(0,0)$. We also discuss a multiplicity results when $f(0,0) = 0 = g(0,0)$.

## 1. Introduction

Consider the boundary-value problem

$$
\begin{gathered}
-\Delta_p u = \lambda f(u,v) \quad \text{in } \Omega \\
-\Delta_q v = \lambda g(u,v) \quad \text{in } \Omega \\
u = 0 = v \quad \text{on } \partial\Omega
\end{gathered}
\tag{1.1}
$$

where $\Delta_s z = \operatorname{div}(|\nabla z|^{s-2}\nabla z)$, $s > 1, \lambda$ is a non-negative parameter, and $\Omega$ is a bounded domain in $\mathbb{R}$ with smooth boundary $\partial\Omega$.

We are interested in the study of positive solutions to (1.1) when no conditions on $f(0,0), g(0,0)$ are assumed, in particular, they could be negative (semipositone systems). Semipositive problems are mathematically challenging area in the study of positive solutions (see [2] and [5]). For a review on semipositone problems, see [3]. In this paper we make the following assumptions:

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(H1) $f, g \in C^1((0,\infty)\times(0,\infty)) \cap C([0,\infty)\times[0,\infty))$ be monotone functions such that $f_u, f_v, g_u, g_v \geq 0$ and $\lim_{u,v\to\infty} f(u,v) = \lim_{u,v\to\infty} g(u,v) = \infty$.

(H2) $\lim_{x\to\infty} \dfrac{f(x, M[g(x,x)]^{1/q-1})}{x^{p-1}} = 0$ for every $M > 0$.

(H3) $\lim_{x\to\infty} \dfrac{g(x,x)}{x^{q-1}} = 0$.

We establish the following existence and multiplicity results:

**Theorem 1.1.** *Let* (H1)–(H3) *hold. Then there exists a positive number $\lambda^*$ such that* (1.1) *has a large positive solution $(u,v)$ for $\lambda > \lambda^*$.*

**Theorem 1.2.** *Let* (H1)–(H3) *hold. Further let $F(s) = f(s, cs)$ and $G(s) = g(\tilde{c}s, s)$ for any $c, \tilde{c} > 0$ and assume that $f$ and $g$ be sufficiently smooth functions in the neighborhood of zero with $F(0) = G(0) = 0$, $F^{(k)}(0) = 0 = G^{(l)}(0)$ for $k = 1, 2, \dots [p-1]$, $l = 1, 2, \dots [q-1]$ where $[s]$ denotes the integer part of $s$. Then* (1.1) *has at least two positive solutions provided $\lambda$ is large.*

This paper extends the recent work in [1], where the authors study such systems with weaker coupling, namely systems of the form,

$$
\begin{gathered}
-\Delta_p u = \lambda_1 \alpha(v) + \mu_1 \delta(u) \quad \text{in } \Omega \\
-\Delta_q v = \lambda_2 \beta(u) + \mu_2 \gamma(v) \quad \text{in } \Omega \\
u = 0 = v \quad \text{on } \partial\Omega
\end{gathered}
\tag{1.2}
$$

where $\lambda_1, \lambda_2, \mu_1$ and $\mu_2$ are non-negative parameters, with the following conditions:

(C1) $\alpha, \beta, \delta, \gamma \in C^1(0,\infty) \cap C[0,\infty)$ be monotone functions such that

$$
\lim_{x\to\infty} \alpha(x) = \lim_{x\to\infty} \beta(x) = \lim_{x\to\infty} \delta(x) = \lim_{x\to\infty} \gamma(x) = \infty.
$$

(C2) $\lim_{x\to\infty} \dfrac{\alpha(M[\beta(x)]^{1/q-1})}{x^{p-1}} = 0$ for every $M > 0$.

(C3) $\lim_{x\to\infty} \dfrac{\delta(x)}{x^{p-1}} = \lim_{x\to\infty} \dfrac{\gamma(x)}{x^{q-1}} = 0$.

In [1], authors establish an existence result for the system (1.2) when $\lambda_1 + \mu_1$ and $\lambda_2 + \mu_2$ are large. In addition, for the case when $f(0) = h(0) = g(0) = \gamma(0) = 0$, authors discuss a multiplicity result for $\lambda_1 + \mu_1$ and $\lambda_2 + \mu_2$ large. Here we extend this study to classes of systems with much stronger coupling. Our approach is based on the method of sub-and supersolutions (see e.g. [4]). In Section 2, we will prove Theorem 1.1, in Section 3, we will prove Theorem 1.2 and in Section 4, we discuss some examples with strong coupling.

## 2. Proof of Theorem 1.1

We extend $f(u,v)$ and $g(u,v)$ for all $(u,v) \in \mathbb{R}^2$ smoothly such that there exists a constant $k_0 > 0$ such that $f(u,v), g(u,v) \geq -k_0$ for all $(u,v) \in \mathbb{R}^2$. We shall establish Theorem 1.1 by constructing a positive weak subsolution $(\psi_1, \psi_2) \in W^{1,p}(\Omega) \cap C(\overline{\Omega}) \times W^{1,q}(\Omega) \cap C(\overline{\Omega})$ and a supersolution $(z_1, z_2) \in W^{1,p}(\Omega) \cap C(\overline{\Omega}) \times W^{1,q}(\Omega) \cap C(\overline{\Omega})$ of (1.1) such that $\psi_i \leq z_i$ for $i = 1, 2$. That is, $\psi_i, z_i$ satisfies

$(\psi_1,\psi_2)=(0,0)=(z_1,z_2)$ on $\partial\Omega$,

$$\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla\xi\,dx \le \lambda\int_\Omega f(\psi_1,\psi_2)\xi\,dx,$$
$$\int_\Omega |\nabla\psi_2|^{p-2}\nabla\psi_2\cdot\nabla\xi\,dx \le \lambda\int_\Omega g(\psi_1,\psi_2)\xi\,dx,$$
$$\int_\Omega |\nabla z_1|^{p-2}\nabla z_1\cdot\nabla\xi\,dx \ge \lambda\int_\Omega f(z_1,z_2)\xi\,dx,$$
$$\int_\Omega |\nabla z_2|^{p-2}\nabla z_2\cdot\nabla\xi\,dx \ge \lambda\int_\Omega g(z_1,z_2)\xi\,dx$$

for all $\xi\in W:=\{\eta\in C_0^\infty(\Omega):\eta\ge 0\text{ in }\Omega\}$.

Let $\lambda_1^{(r)}$ the first eigenvalue of $-\Delta_r$ with Dirichlet boundary conditions and $\phi_r$ the corresponding eigenfunction with $\phi_r>0;\Omega$ and $\|\phi_r\|_\infty=1$ for $r=p,q$. Let $m,\delta>0$ be such that $|\nabla\phi_r|^r-\lambda_1^{(r)}\phi_r^r\ge m$ on $\overline{\Omega}_\delta=\{x\in\Omega|d(x,\partial\Omega)\le\delta\}$ for $r=p,q$. (This is possible since $|\nabla\phi_r|\ne 0$ on $\partial\Omega$ while $\phi_r=0$ on $\partial\Omega$ for $r=p,q$). We shall verify that

$$(\psi_1,\psi_2):=\Big([\frac{\lambda k_0}{m}]^{1/p-1}(\frac{p-1}{p})\phi_p^{p/p-1},[\frac{\lambda k_0}{m}]^{1/q-1}(\frac{q-1}{q})\phi_q^{q/q-1}\Big),$$

is a subsolution of (1.1) for $\lambda$ large. Let $\xi\in W$. Then

$$\begin{aligned}\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla\xi\,dx &= \Big(\frac{\lambda k_0}{m}\Big)\int_\Omega \phi_p|\nabla\phi_p|^{p-2}\nabla\phi_p\cdot\nabla\xi\,dx\\ &= \Big(\frac{\lambda k_0}{m}\Big)\Big\{\int_\Omega |\nabla\phi_p|^{p-2}\nabla\phi_p\cdot\nabla(\phi_p\xi)\,dx-\int_\Omega|\nabla\phi_p|^p\xi\,dx\Big\}\\ &= \Big(\frac{\lambda k_0}{m}\Big)\Big\{\int_\Omega[\lambda_1^{(p)}\phi_p^p-|\nabla\phi_p|^p]\xi\,dx\Big\}.\end{aligned}$$

Similarly

$$\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla\xi\,dx=\Big(\frac{\lambda k_0}{m}\Big)\Big\{\int_\Omega[\lambda_1^{(q)}\phi_q^q-|\nabla\phi_q|^q]\xi\,dx\Big\}.$$

Now on $\overline{\Omega}_\delta$ we have $|\nabla\phi_r|^r-\lambda_1^{(s)}\phi_r^r\ge m$ for $r=p,q$. Which implies that

$$\frac{k_0}{m}\Big(\lambda_1^{(p)}\phi_p^p-|\nabla\phi_p|^p\Big)-f(\psi_1,\psi_2)\le 0,$$
$$\frac{k_0}{m}\Big(\lambda_1^{(q)}\phi_q^q-|\nabla\phi_q|^q\Big)-g(\psi_1,\psi_2)\le 0.$$

Next on $\Omega-\overline{\Omega}_\delta$ we have $\phi_p\ge\mu$, $\phi_q\ge\mu$ for some $\mu>0$, and therefore for $\lambda$ large

$$f(\psi_1,\psi_2)\ge\frac{k_0}{m}\lambda_1^{(p)}\ge\frac{k_0}{m}\lambda_1^{(p)}\phi_p^p-|\nabla\phi_p|^p,$$
$$g(\psi_1,\psi_2)\ge\frac{k_0}{m}\lambda_1^{(q)}\ge\frac{k_0}{m}\lambda_1^{(q)}\phi_q^q-|\nabla\phi_q|^q.$$

Hence

$$\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla\xi\,dx \le \lambda\int_\Omega f(\psi_1,\psi_2)\xi\,dx,$$
$$\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla\xi\,dx \le \lambda\int_\Omega g(\psi_1,\psi_2)\xi\,dx;$$

i.e., $(\psi_1,\psi_2)$ is a subsolution of (1.1) for $\lambda$ large.

Next let $e_r$ be the solution of $-\Delta_r e_r = 1$ in $\Omega$, $e_r = 0$ on $\partial\Omega$ for $r = p, q$. Let $(z_1, z_2) := \Big(\frac{c}{\mu_p}\lambda^{1/p-1}e_p, [g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}\lambda^{1/q-1}e_q\Big)$ where $\mu_r = \|e_r\|_\infty$; $r = p, q$. Then

$$\begin{aligned}\int_\Omega |\nabla z_1|^{p-2}\nabla z_1\cdot\nabla\xi\,dx &= \lambda\big(\frac{c}{\mu_p}\big)^{p-1}\int_\Omega |\nabla e_p|^{p-2}\nabla e_p\cdot\nabla\xi\,dx\\ &= \frac{1}{(\mu_p)^{p-1}}(c\lambda^{1/p-1})^{p-1}\int_\Omega \xi\,dx.\end{aligned}$$

By (H2) we can choose $c$ large enough so that

$$\begin{aligned}&\frac{1}{(\mu_p)^{p-1}}(c\lambda^{1/p-1})^{p-1}\int_\Omega \xi\,dx\\ &\geq \lambda\int_\Omega f(c\lambda^{1/p-1}, [g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}\lambda^{1/q-1}\mu_q)\xi\,dx\\ &\geq \lambda\int_\Omega f(c\lambda^{1/p-1}\frac{e_p}{\mu_p}, [g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}\lambda^{1/q-1}e_q)\xi\,dx\\ &= \lambda\int_\Omega f(z_1,z_2)\xi\,dx.\end{aligned}$$

Next

$$\begin{aligned}\int_\Omega |\nabla z_2|^{q-2}\nabla z_2\cdot\nabla\xi\,dx &= \lambda[g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]\int_\Omega |\nabla e_q|^{q-2}\nabla e_q\cdot\nabla\xi\,dx\\ &= \lambda[g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]\int_\Omega \xi\,dx\end{aligned}$$

By (H3) choose $c$ large so that $\frac{1}{\lambda^{1/q-1}}\mu_q \geq \frac{[g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}}{c\lambda^{1/p-1}}$, then

$$\begin{aligned}&\lambda[g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]\int_\Omega \xi\,dx\\ &\geq \lambda\int_\Omega g\big(c\lambda^{1/p-1}, [g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}\lambda^{1/q-1}\mu_q\big)\xi\,dx\\ &\geq \lambda\int_\Omega g\big(c\lambda^{1/p-1}\frac{e_p}{\mu_p}, [g(c\lambda^{1/p-1}, c\lambda^{1/p-1})]^{1/q-1}\lambda^{1/q-1}e_q\big)\xi\,dx\\ &= \lambda\int_\Omega g(z_1,z_2)\xi\,dx;\end{aligned}$$

i.e., $(z_1, z_2)$ is a supersolution of (1.1) with $z_i \geq \psi_i$ for $c$ large, $i = 1, 2$. (Note $|\nabla e_r| \neq 0; \partial\Omega$ for $r = p, q$).

Thus, there exists a solution $(u, v)$ of (1.1) with $\psi_1 \leq u \leq z_1, \psi_2 \leq v \leq z_2$. This completes the proof of Theorem 1.1.

## 3. Proof of Theorem 1.2

To prove Theorem 1.2, we will construct a subsolution $(\psi_1,\psi_2)$, a strict supersolution $(\zeta_1,\zeta_2)$, a strict subsolution $(w_1, w_2)$, and a supersolution $(z_1, z_2)$ for (1.1) such that $(\psi_1,\psi_2) \leq (\zeta_1,\zeta_2) \leq (z_1, z_2)$, $(\psi_1,\psi_2) \leq (w_1, w_2) \leq (z_1, z_2)$, and

$(w_1, w_2) \not\leq (\zeta_1, \zeta_2)$. Then (1.1) has at least three distinct solutions $(u_i, v_i)$, $i = 1, 2, 3$, such that $(u_1, v_1) \in [(\psi_1, \psi_2), (\zeta_1, \zeta_2)]$, $(u_2, v_2) \in [(w_1, w_2), (z_1, z_2)]$, and

$$(u_3, v_3) \in [(\psi_1, \psi_2), (z_1, z_2)] \setminus \big([(\psi_1, \psi_2), (\zeta_1, \zeta_2)] \cup [(w_1, w_2), (z_1, z_2)]\big).$$

We first note that $(\psi_1, \psi_2) = (0, 0)$ is a solution (hence a subsolution). As in Section 2, we can always construct a large supersolution $(z_1, z_2)$. We next consider

$$\begin{gathered} -\Delta_p w_1 = \lambda \tilde{f}(w_1, w_2) \quad \text{in } \Omega \\ -\Delta_q w_2 = \lambda \tilde{g}(w_1, w_2) \quad \text{in } \Omega \\ w_1 = 0 = w_2 \quad \text{on } \partial\Omega \end{gathered} \tag{3.1}$$

where $\tilde{f}(u, v) = f(u, v) - 1$ and $\tilde{g}(u, v) = g(u, v) - 1$. Then by Theorem 1.1, (3.1) has a positive solution $(w_1, w_2)$ when $\lambda$ is large. Clearly this $(w_1, w_2)$ is a strict subsolution of (1.1). Finally we construct the strict supersolution $(\zeta_1, \zeta_2)$.

To do so, we let $\phi_p, \phi_q$ as described in Section 2. We note that there exists positive constants $c_1$ and $c_2$ such that

$$\phi_p \leq c_1 \phi_q \quad \text{and} \quad \phi_q \leq c_2 \phi_p. \tag{3.2}$$

Let $(\zeta_1, \zeta_2) = (\epsilon\phi_p, \epsilon\phi_q)$ where $\epsilon > 0$. Let $H_p(s) := \lambda_1^{(p)} s^{p-1} - \lambda f(s, c_2 s)$ and $H_q(s) := \lambda_1^{(q)} s^{q-1} - \lambda g(c_1 s, s)$. Observe that $H_p(0) = H_q(0) = 0$, $H_p^{(k)}(0) = 0 = H_q^{(l)}(0)$ for $k = 1, 2, \dots [p-2]$ and $l = 1, 2, \dots [q-2]$. $H_p^{(p-1)}(0) > 0$ and $H_q^{(q-1)}(0) > 0$ if $p, q$ are integers, while $\lim_{r \to 0} H^{([p])}(r) = +\infty = \lim_{r \to 0} H^{([q])}(r)$ if $p, q$ are not integers. Thus there exists $\theta$ such that $H_p(s) > 0$ and $H_q(s) > 0$ for $s \in (0, \theta]$. Hence for $0 < \epsilon \leq \theta$ we have

$$\begin{aligned} \lambda_1^{(p)} (\zeta_1)^{p-1} = \lambda_1^{(p)} (\epsilon\phi_p)^{p-1} &> \lambda f(\epsilon\phi_p, c_2 \epsilon\phi_p) \\ &\geq \lambda f(\epsilon\phi_p, \epsilon\phi_q) \\ &= \lambda f(\zeta_1, \zeta_2) \quad x \in \Omega, \end{aligned} \tag{3.3}$$

and similarly we get

$$\begin{aligned} \lambda_1^{(q)} (\zeta_2)^{q-1} = \lambda_1^{(q)} (\epsilon\phi_q)^{q-1} &> \lambda g(c_1 \epsilon\phi_q, \epsilon\phi_q) \\ &\geq \lambda g(\epsilon\phi_p, \epsilon\phi_q) \\ &= \lambda g(\zeta_1, \zeta_2), \quad x \in \Omega. \end{aligned} \tag{3.4}$$

Using the inequalities (3.3) and (3.4) we have,

$$\begin{aligned} \int_\Omega |\nabla\zeta_1|^{p-2} \nabla\zeta_1 \cdot \nabla\xi \, dx = \epsilon^{p-1} \int_\Omega |\nabla\phi_p|^{p-2} \nabla\phi_p \cdot \nabla\xi \\ = \int_\Omega \lambda_1^{(p)} (\epsilon\phi_p)^{p-1} \xi \, dx \\ > \lambda \int_\Omega f(\zeta_1, \zeta_2) \xi \, dx. \end{aligned}$$

Similarly we have

$$\int_\Omega |\nabla\zeta_2|^{q-2} \nabla\zeta_2 \cdot \nabla\xi \, dx > \lambda \int_\Omega g(\zeta_1, \zeta_2) \xi \, dx$$

Thus $(\zeta_1, \zeta_2)$ is a strict supersolution. Here we can choose $\epsilon$ small so that $(w_1, w_2) \not\leq (\zeta_1, \zeta_2)$.

Hence there exists solutions $(u_1, v_1) \in [(\psi_1, \psi_2), (\zeta_1, \zeta_2)]$, $(u_2, v_2) \in [(w_1, w_2), (z_1, z_2)]$, and $(u_3, v_3) \in [(\psi_1, \psi_2), (z_1, z_2)] \setminus \big([(\psi_1, \psi_2), (\zeta_1, \zeta_2)] \cup [(w_1, w_2), (z_1, z_2)]\big)$. Since $(\psi_1, \psi_2) \equiv (0, 0)$ is a solution it may turn out that $(u_1, v_1) \equiv (\psi_1, \psi_2) \equiv (0, 0)$. In any case we have two positive solutions $(u_2, v_2)$ and $(u_3, v_3)$. Hence Theorem 1.2 holds.

**Remark 3.1.** Note that in the construction of the supersolution $(\zeta_1, \zeta_2)$ we require the conditions at zero on $F$ and $G$ only for the constants $c = c_2$ and $\tilde{c} = c_1$.

## 4. Examples

**Example 4.1.** Consider the problem

$$\begin{gathered} -\Delta_p u = \lambda[v^\alpha + (uv)^\beta - 1] \quad \text{in } \Omega \\ -\Delta_q v = \lambda[u^\sigma + (uv)^{\gamma/2} - 1] \quad \text{in } \Omega \\ u = 0 = v \quad \text{on } \partial\Omega \end{gathered} \tag{4.1}$$

where $\alpha, \beta, \sigma, \gamma$ are positive parameters. Then it is easy to see that (4.1) satisfies the hypotheses of Theorem 1.1 if $\max\{\sigma, \gamma\}\frac{\alpha}{q-1} < p - 1$, $(\max\{\sigma, \gamma\}\frac{1}{q-1} + 1)\beta < p - 1$ and $\max\{\sigma, \gamma\} < q - 1$.

**Example 4.2.** Let

$$h(x) = \begin{cases} x^\alpha; & x \le 1 \\ \frac{\alpha}{\sigma}x^\sigma + (1 - \frac{\alpha}{\sigma}); & x > 1, \end{cases} \quad \text{and} \quad \gamma(x) = \begin{cases} x^\mu; & x \le 1 \\ \frac{\mu}{\delta}x^\delta + (1 - \frac{\mu}{\delta}); & x > 1, \end{cases}$$

where $\alpha, \sigma, \mu, \delta$ are positive parameters. Here we assume $\alpha > p-1$ if $p$ is an integer, $\alpha > [p]$ if $p$ is not an integer, $\mu > q - 1$ if $q$ is an integer and $\mu > [q]$ if $q$ is not an integer.

Consider the problem

$$\begin{gathered} -\Delta u = \lambda[1 + u^\beta]h(v) \quad \text{in } \Omega \\ -\Delta v = \lambda\gamma(u) \quad \text{in } \Omega \\ u = 0 = v \quad \text{on } \partial\Omega \end{gathered} \tag{4.2}$$

where $0 \le \beta < p - 1$. Then it is easy to see that (4.2) satisfies the hypotheses of Theorem 1.2 if $\delta\sigma < [p - 1 - \beta](q - 1)$ and $\delta < q - 1$.

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