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MATHEMATICS AND FISHERIES: MATCH OR MISMATCH?

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Abstract. Mathematics plays a major role in contemporary fisheries management. Stock assessments often depend on elaborate models used to set catch levels and address other policy objectives. In recent years, the collapse of various important fish stocks has caused some critics to suggest that mathematical models actually obscure the truth by narrowing scientific understanding to the realm of quantifiable events. In the words of one fisherman, "Mathematics has highjacked the definition and position of real science." In this paper, I present a number of typical fishery models, examine their limitations, discuss controversies about their use, and explore possible alternatives. I draw on examples from economics and investment theory to illustrate the problem of making credible predictions about an uncertain future. The constraints of the real world, where people care deeply about policy consequences, have altered my scientific perspective as an applied mathematician. This paper reflects the evolution of thought that has accompanied my experience working for 28 years at the Pacific Biological Station in Nanaimo, B.C., the host city for this conference.

1. Introduction

Mathematics offers a wonderful tool for speculation about biological principles that govern animal populations. Most sensible rules for birth, growth, movement, and death can be stated mathematically. Ideally, theoreticians can use these assumptions to derive theorems that characterize model behavior. If theory seems intractable, computer simulations allow scientists to explore a model's properties. Unfortunately, even the most brilliant analysis leaves a nagging question unresolved. Does this set of rules adequately describe how the world actually works?

Fishery science often depends heavily on mathematical methods for inferring the state of fish stocks from limited available data. Fishermen care about fish, not theorems, and they can be highly critical of models that fail to capture the biological world as they see it. An optimistic theory that predicts a high birth rate becomes meaningless if, in fact, these births do not take place. Similarly, a pessimistic forecast that proves wrong can also bring mathematical models into disrepute. Speaking in the context of a collapsed fishery, the fisherman James O'Malley [16] concluded that "in our attempt to comprehend the oceans ... mathematics has

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been elevated to a status which suppresses knowledge and actually detracts from our efforts to acquire knowledge."

In this paper, I examine the role of mathematics as a tool for making decisions about the real world. Case studies from economics and fishery management suggest reasons to apply mathematics with caution. Statistics plays a key role in financial and fishery models, where observed data never conform to a deterministic model. In practice, the interpretation of a data set can vary greatly with the choice of assumptions about statistical error. A detailed analysis of one fish stock illustrates the process commonly used to estimate the unknown total biomass from available data. I conclude by discussing a recent shift in fishery research from estimation methods to robust decision algorithms that perform well across a broad spectrum of possible models.

2. Investments and fisheries

In 1973, a new mathematical formula (Black and Scholes [1]; Merton [14]) revolutionized the world of finance by providing a rational system for setting prices in the options market. Essentially, an option contract gives its owner the right, without obligation, to buy or sell an asset at a specified future time and price. A *call* option gives the right to buy, whereas a *put* option gives the right to sell. In practice, traders want to know how to assign value to such contracts. For example, if a stock has current price S, what is a fair price C for a call option to buy the stock at the future time T and specified price K (the so-called *strike* price)? If the prevailing interest rate is r and the stock price moves log-normally in continuous time with standard deviation σ (a Wiener process), then the famous Black-Scholes formula gives the call price

$$C(T,S,K,r,\sigma) = S\,\Phi\Big(x + \frac{\sigma\sqrt{T}}{2}\Big) - K\,e^{-rT}\,\Phi\Big(x - \frac{\sigma\sqrt{T}}{2}\Big)\,, \tag{2.1}$$

where

$$x = \frac{rT + \log\frac{S}{K}}{\sigma\sqrt{T}}$$

and $\Phi(\cdot)$ denotes the cumulative normal distribution function

$$\Phi(y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-x^2/2}dx.$$

The Internet (e.g., [23]) provides much more information and various mathematical proofs.

The formula (2.1) solves an important problem in investment theory and has the practical advantage that it can be implemented easily on the trading floor. Mathematically, it follows from a continuous-time stochastic model. In 1997, after Black had died, Scholes and Merton won the Nobel prize in economics "for a new method to determine the value of derivatives", including this result. Not surprisingly, investment strategists looked for opportunities posed by such an elegant theory.

Lowenstein [12] gives an engaging account of the most famous example, Long-Term Capital Management (LTCM). The name "hedge fund" derives from the expression "to hedge one's bets", i.e., to bound risk like a common hedge bounds a garden [12, p. 25]. A hedge fund typically bets on the *spread* between current and future prices, where the future option usually sells at a discount and the spread

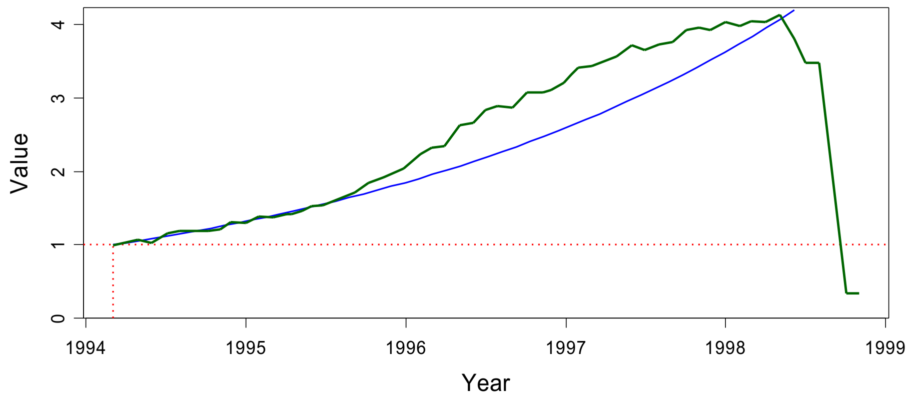

FIGURE 1. Gross value of \$1 invested in Long-Term Capital Management, March 1994 to October 1998 (green line; data from a graph by Lowenstein [12]). The blue curve represents an exponential growth rate that compounds to 40% per year (proportional to 1.4^t with time t in years).

shrinks over time. In this scenario, an investor is protected from the risk of market fluctuations because the two prices generally move in tandem while the spread between them declines.

In February 1994, armed with a powerful mathematical theory and supported by the now-famous economists Merton and Scholes, the experienced investment manager John Merriweather started LTCM with assets of \$1.25 billion contributed by wealthy investors [12, p. 39]. For about four years following its inception, the investment grew at an astonishing rate that kept it above a steady exponential growth curve of 40% annually (Figure 1). Managers achieved this growth using highly leveraged betting on assets purchased with loans from major banks. The fund's capital value, \$4.7 billion at the start of 1998, had to support borrowed assets of about \$100 billion [12, p. xix–xx]. Furthermore, in Lowenstein's words, LTCM "had entered into thousands of derivative contracts, which had endlessly intertwined it with every bank on Wall Street. These contracts, essentially side bets on market prices, covered an astronomical sum – more than \$1 trillion worth of exposure."

According to the model that underlies (2.1), asset values should move lognormally with a standard deviation $\sigma\sqrt{\delta t}$ during a small time period δt. Theoretically, a huge change in a short time should be nearly impossible, but extreme market conditions sometimes cause the model to fail. On August 17, 1998, Russia devalued the ruble and declared a moratorium on its Treasury debt of about \$13.5 billion [8]. That event and other market crises caused LTCM's equity to drop much more rapidly than the most extreme model predictions. On September 23, 1998, it appeared that the fund might not survive another day [12, p. xx]. Faced with the prospect that a default by LTCM might trigger market panic and collapse, the Federal Reserve Bank of New York organized a bail-out loan of \$3.6 billion. A consortium of 14 major firms now had the power to oversee all fund transactions. The initial investors, including Merriweather, Merton, and Scholes, lost millions of

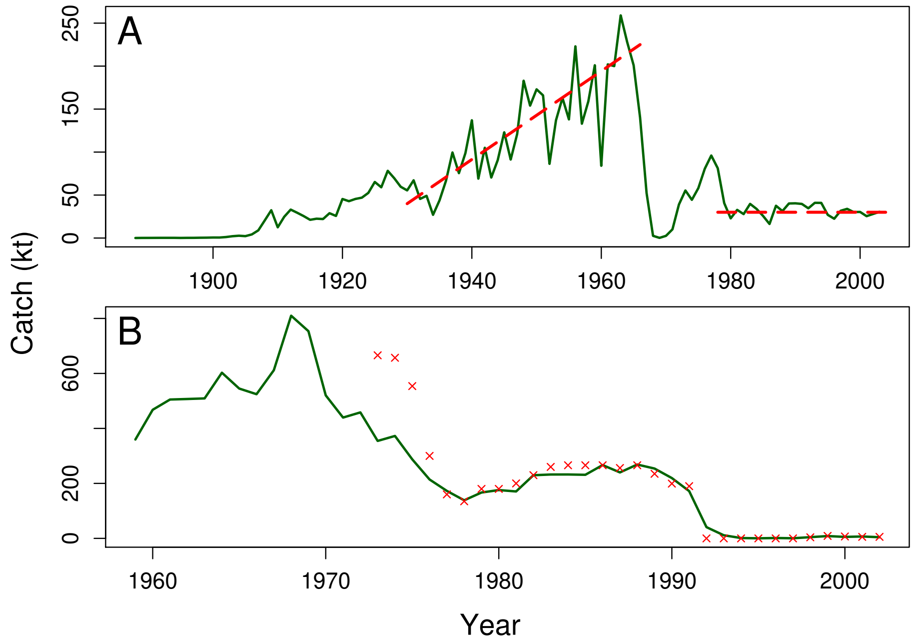

FIGURE 2. Annual catch (kt = 10^6 kg) from two Canadian fisheries. **A**. British Columbia herring (*Clupea pallasi*), where red lines highlight two periods: (1930-1966) historical expansion along the Pacific coast and (1978-present) current stable fishery with an annual catch near 30 kt. Data from Jake Schweigert, Pacific Biological Station, Nanaimo, B.C. **B**. Newfoundland northern cod (*Gadus morhua*) in area 2J/3KL, where red 'x' symbols show annual quotas imposed since 1973. Data from [11, Table 1, p. 63–64].

dollars. LTCM repaid its loans by December, 1999, and quietly closed down a few weeks later [15].

This cautionary tale gives applied mathematicians ample opportunity to speculate about the application of theory to real world problems. Lowenstein [12] paints a vivid picture of the players involved, and two related TV programs ([9], [15]) give us an opportunity to hear their views. For example, Myron Scholes speculated [15] that the difficulties with LTCM didn't come only from the models. "It could be inputs to the models, it could be the models themselves, it could be a combination of many things. And so just saying models were flawed is not necessarily the right answer." Paul Samuelson, winner of the 1970 Economics Nobel Prize for his analytical work in the field, observed [15] that "There is a tempting and fatal fascination in mathematics. Albert Einstein warned against it. He said elegance is for tailors, don't believe in something because it's a beautiful formula. There will always be room for judgment." U.S. Federal Reserve Chairman Alan Greenspan asked [15] "How much dependence should be placed on financial modeling, which for all its sophistication can get too far ahead of human judgment?" Norbert Wiener, who invented the continuous-time stochastic process that bears his name and underlies the Black-Scholes formula, regarded skepticism as a professional obligation [22]: "One of the chief duties of a mathematician in acting as an advisor to scientists is to discourage them from expecting too much of mathematicians."

The shared word *stock* curiously links financial and fishery management. In the investment world, a stock has a current price S known from the marketplace itself. By contrast, a fish stock has a current biomass B that generally is not known and may be impossible to measure directly. Consequently, biological models to predict future values B have even greater uncertainty than financial models for predicting S. Like financial managers who must decide how much stock to buy, sell, or hold, fishery managers must decide how much stock biomass to allow as catch. The same urge to use quantitative methods prevails in both worlds. Unfortunately, like financial models, fishery management models can fail to represent the world adequately.

Figure 2 tells the tale of two Canadian fisheries, one successfully managed for recovery and the other not. In the first case (Figure 2A), the British Columbia herring fishery experienced a historical period of expansion, followed by severe regulations to permit stock recovery. For the last two decades, under restrictive management, the fishery has sustained a steady catch at a fraction of historical levels. Because higher annual catches might only drive down market prices, the industry finds economic reasons to agree with current regulations. Furthermore, herring provide food for other commercial fish species, and stakeholders generally recognize the importance of a robust herring population for other Pacific fisheries.

In the second case (Figure 2B), the Newfoundland cod fishery also experienced a historical period of expansion, followed by quotas to limit the catch. However, seemingly moderate catch levels during the 1980s still did not allow the population to recover, and the fishery was closed in 1992. Evidence suggests that the population still hasn't recovered enough to allow more than very low levels of catch. Lilly et al. [11] discuss possible reasons (still controversial) for this fishery collapse. Why couldn't historical catch levels be maintained? Why hasn't the stock recovered like Pacific herring? According to one scenario, fishery management did not respond quickly enough to curtail a fishery on a population becoming stressed by changing ocean conditions in the north Atlantic.

Some people saw the problem coming and advised caution, just as some economists had expressed concerns for the levels of risk prevailing at LTCM near the end of 1997. But fishery and financial management always entail risk, and it's tempting to suppose that things won't change too dramatically in a short time. Rapid climate change, like a Russian default on bonds, may not come up on the radar screen until it's too late. Mathematical models, conditioned to past performance, tend to handle regime shifts poorly. Myron Scholes expressed frustration with this problem at LTCM [15]: "In August of 1998, after the Russian default, ... all the relations that tended to exist in a recent past seemed to disappear."

3. Fishery models

Like economists seeking formulas for option prices, fishery scientists try to find sensible rules for setting catch quotas. Figure 3 shows the logical structure of a typical fishery model. An unknown biomass follows a dynamic process governed by unknown parameters and the removal of a known catch. Natural variability enters the dynamics as stochastic process error. Other known data come from

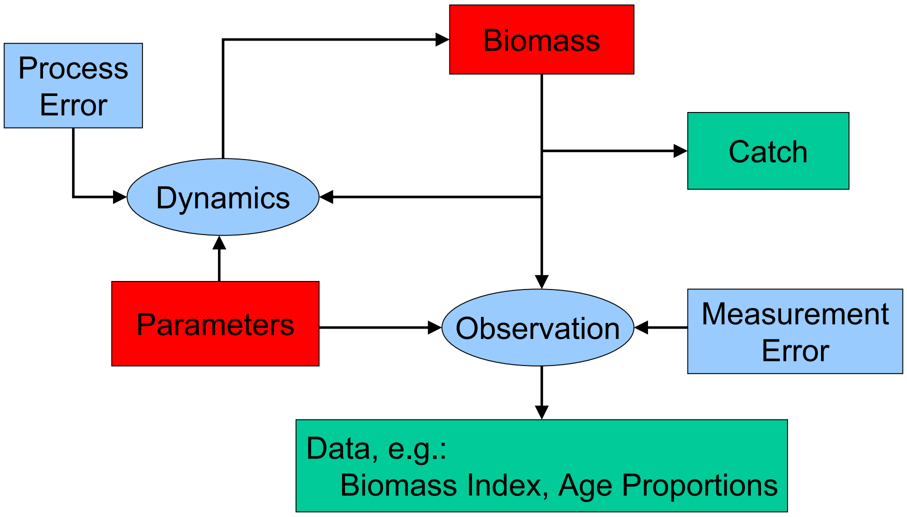

FIGURE 3. A stochastic, dynamic fishery model. Colors indicate the role of model components: theory (blue), observed data (green), and unknown quantities (red).

biological observations, such as surveys and samples of the catch. An observation model connects these data to the biomass, subject to measurement error, which introduces another source of statistical noise. Practical applications depend on a key idea related to the color codes in Figure 3: **Use the blue theory to estimate the unknown red quantities from the known green observations.**

The simple deterministic model in Table 1 illustrates how this procedure might actually work. Imagine a fixed stock of fish altered only by the removal of catch C_t at various times t. Let B_t denote the biomass available prior to the catch C_t, and suppose that a survey method exists to measure a biomass index I_t proportional to B_t through an unknown coefficient q. (For example, the survey might use acoustic sensors to produce a signal proportional to biomass.) The index data I_t at times $t = 1, 2$ correspond to the two observation equations (T1.2)–(T1.3), and the transition of biomass B_t from $t = 1$ to $t = 2$ gives the dynamic equation (T1.1). In this example, only a few mathematical symbols define the dynamic ($=$,$-$) and observation ($=$,$\times$) models. Three equations relate three unknowns (B_1,B_2,q) to three observations (C_1,I_1,I_2). Simple algebra gives the estimates (T1.4)–(T1.6) of unknown reds in terms of known greens.

The depletion model in Table 1 can readily be extended to include multiple time steps $t = 1, \dots, n$, and the equations then imply the general result

$$I_t = qB_1 - q\sum_{i=1}^{t-1} C_i \tag{3.1}$$

for each observation I_t, where by definition $\sum_{i=1}^{0} C_i = 0$ when $t = 1$. In this formulation, the model has only two unknowns (q,B_1), regardless of the number n

TABLE 1. A simple fishery depletion model with deterministic equations for dynamics and measurement. Three data values (catch C_1 and biomass indices I_1, I_2) determine three unknowns (parameter q and biomass levels B_1, B_2).

$$\text{Dynamics:} \quad B_2 = B_1 - C_1 \tag{T1.1}$$

$$\text{Observations:} \quad I_1 = q \times B_1 \tag{T1.2}$$

$$I_2 = q \times B_2 \tag{T1.3}$$

$$\text{Estimates:} \quad q = \frac{I_1 - I_2}{C_1} \tag{T1.4}$$

$$B_1 = \frac{I_1}{I_1 - I_2} C_1 \tag{T1.5}$$

$$B_2 = \frac{I_2}{I_1 - I_2} C_1 \tag{T1.6}$$

of observed time steps. An estimate of B_1 gives biomass estimates

$$B_t = B_1 - \sum_{i=1}^{t-1} C_i \tag{3.2}$$

for all future times $t > 1$. The overdetermined system (3.1) of n equations with two unknowns cries out for statistical analysis, such as linear regression of I_t on the cumulative catch $\sum_{i=1}^{t-1} C_i$. But exactly how should we introduce error into the model that underlies (3.1)? If we think that immigration and emigration randomly alter the biomass B_t, we could introduce process error into the dynamics $B_{t+1} = B_t - C_t$. If we think that our measurements I_t aren't exactly proportional to B_t, we could (and almost certainly should) introduce measurement error into the observation equation $I_t = qB_t$. Such choices correspond to the decision by Black, Scholes, and Merton to use a Wiener process for modeling stock price fluctuations. In some cases, different choices can dramatically alter the analysis.

Figure 4 illustrates this issue for the problem of fitting a straight line through four data points that lie on the corners of a rectangle in the xy-plane [19]. The estimated regression line is horizontal if the error occurs in y, but vertical if the error occurs in x. Because a line must have slope somewhere between 0 (horizontal) and ∞ (vertical), this example shows that the perceived signal depends entirely on the definition of noise. By analogy, think of watching 3D movies with polaroid glasses. The two polarized axes define separate images for the left and right eyes. Noise for the right eye is signal for the left, and vice-versa. Seemingly innocent choices of error in biological models sometimes disguise other interpretations of the data, even given the same deterministic model (e.g., a straight line in Figure 4).

Realistic fishery models involve much more complexity and data than the simple prototypes (T1.1)–(T1.3) or (3.1)–(3.2). For example, Figure 5 shows results obtained from a model of a particular stock of Pacific ocean perch (POP) on the coast of British Columbia, Canada [20]. The analysis uses three main data sets:

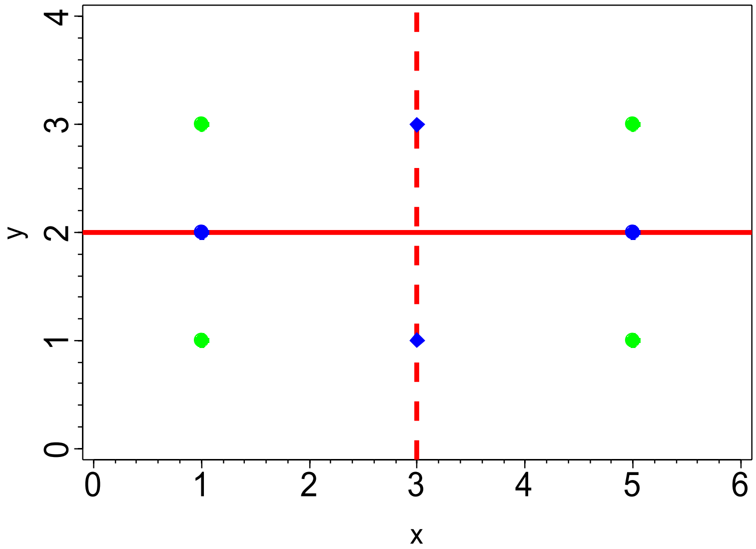

FIGURE 4. Schnute's [19, Figure 1] demonstration of the importance of noise in stochastic models. Four observed data points (green circles) lie at the corners of a rectangle. If error occurs in the y coordinate, the data determine a horizontal regression line (solid red) through two observed mean values $y = 2$ (blue circles). If error occurs in the x coordinate, the data determine a vertical regression line (broken red) through two observed mean values $x = 3$ (blue diamonds).

the annual catch C_t, intermittent biomass index measurements I_{tj} obtained in various years t by two methods ($j = 1, 2$), and sample proportions p_{at} of fish in the catch that have age a in year t. Historical data also give reasonable values for the weight w_a of a fish at age a. POP start to appear in the fishery at recruitment age $k = 7$, although the data suggest that young fish are less vulnerable to the gear than older fish. The model tries to capture this phenomenon with unknown selectivity coefficients β_a that increase steadily toward 1 as $a \to \infty$.

Internally, the model keeps track of the number N_{at} of fish at age a in year t prior to the fishery. From this matrix, various annual population characteristics can be computed, including

$$\text{recruitment:} \quad R_t = N_{kt}\,, \tag{3.3}$$

$$\text{total biomass:} \quad B'_t = \sum_{a \geq k} w_a N_{at}\,, \tag{3.4}$$

$$\text{selected biomass:} \quad B_t = \sum_{a \geq k} \beta_a w_a N_{at}\,, \tag{3.5}$$

$$\text{selected proportion:} \quad u_{at} = \beta_a N_{at} \Big/ \sum_{a \geq k} \beta_a N_{at}\,. \tag{3.6}$$

Given suitable definitions of measurement error, the index data I_{tj} should be proportional to the selected biomass B_t with unknown coefficients q_j ($j = 1, 2$), and

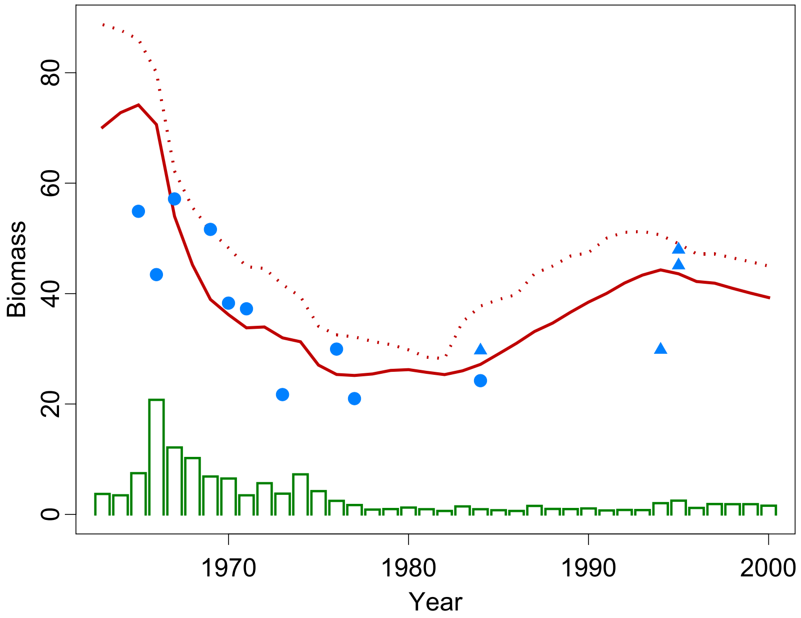

FIGURE 5. Annual estimates of biomass (kt $= 10^6$ kg) for a stock of Pacific ocean perch (POP, *Sebastes alutus*) assessed in [20]. The model uses historical data on catch (kt, green bars) and two survey indices with distinct coefficients q_j ($j = 1, 2$) in (3.7). Observed index values I_{tj} are scaled to biomass levels I_{tj}/q_j for $j = 1$ (blue circles) and $j = 2$ (blue triangles). The biomass vulnerable to the fishery (solid red line, B_t in (3.5)) is smaller than the total biomass (dotted line, B'_t in (3.4)), a feature represented in the model by selectivity coefficients β_a that increase steadily to 1 with increasing fish age a.

the observed proportions p_{at} should match the internal proportions u_{at}. Thus, the model's observation component (Figure 3) involves stochastic counterparts of the deterministic equations

$$I_{tj} = q_j B_t \,, \tag{3.7}$$

$$p_{at} = u_{at} \,. \tag{3.8}$$

Figure 5 shows a fishery similar to the one portrayed in Figure 2A. Large historical catches C_t have been curtailed by regulation to a relatively modest steady catch during the last two decades. According to the model, historically large biomass levels B_t were driven down by large catches in the 1960s and 1970s, but have recovered somewhat after the catch was reduced. Sporadic surveys lend support to this scenario, but the survey index data deviate substantially from the biomass trend. (Note the scatter of blue points around the solid red line.) A recent slow decline in biomass stems from low recruitment in the 1990s, as discussed below.

The observed age data p_{at}, portrayed as a *bubble plot* [17] in Figure 6A, display a pattern with several diagonal lines of large bubbles. These correspond to episodes of high recruitment; for example, a particularly strong cohort proceeds from ages

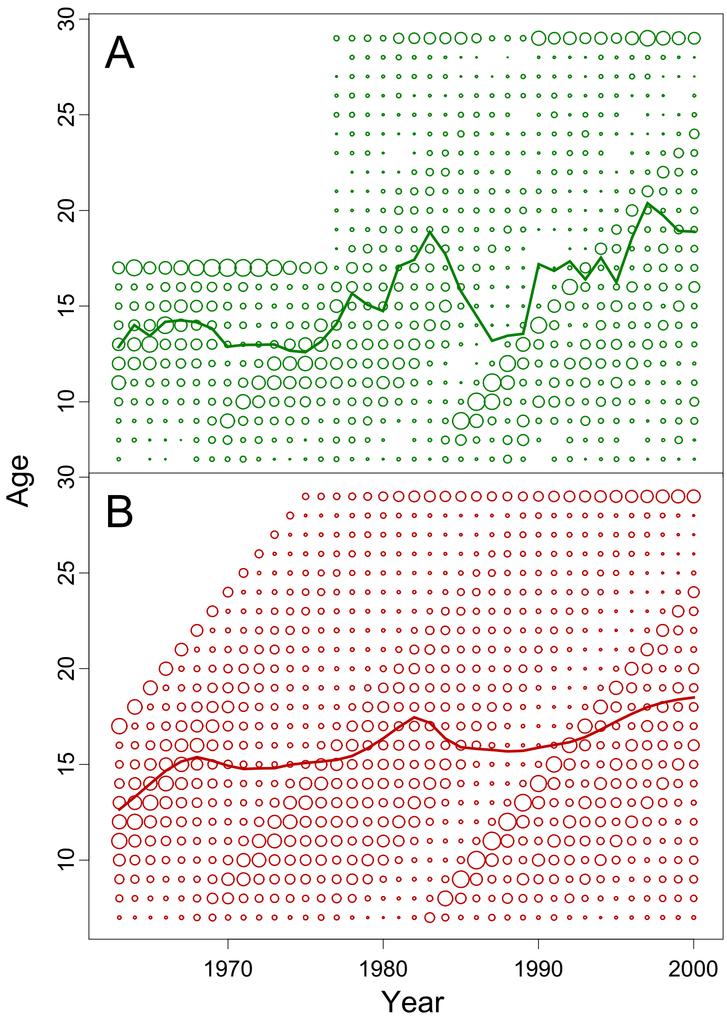

FIGURE 6. (A) Observed and (B) estimated age distributions p_{at} and u_{at} of the population in Figure 5. Circular areas represent relative age proportions within each year, where $\sum_a p_{at} = \sum_a u_{at} = 1$ for each t. Solid lines show the annual mean age. The circle at the top of each column represents a "plus class", i.e., fish of the indicated age or older. An improved ageing method in 1977 gave better discrimination to ages in the range 17 to 29. Observed data in (A) show an abrupt increment in age resolution. Model estimates in (B) show a more gradual change as the plus class increases annually from age 17 in 1963.

7 to 24 during the years 1983–2000. POP live long enough to spawn many times. Ocean conditions and other factors determine the success of each annual spawning event, which occasionally pays off very well. Like good financiers, the fish distribute their genetic investments across time, taking advantage of occasional high

returns. The estimated proportions u_{at} show this cohort effect even more strongly (Figure 6B). Each diagonal has small proportions of fish near the recruitment age $k = 7$, due to low selectivity by the fishery. Natural and fishing mortality also cause the proportions to decline as fish reach advanced ages.

For brevity, I have omitted many technical details in this discussion of the POP example. The model handles process error (Figure 3) by allowing independent recruitments R_t for each cohort, with some constraints on their variability and serial correlation. Stochastic versions of (3.7)–(3.8) introduce measurement error. As in the dilemma of Figure 4, model definition here requires balancing recruitment variability with survey observation error. The scenario portrayed in Figures 5–6 involves estimates of nearly 60 independent unknown quantities from which many others (such as the matrices N_{at} and u_{at}) are calculated. I haven't even mentioned the statistical properties of all these estimates, which might be assessed using computer-intensive algorithms from Bayesian statistics. Interested readers can find a complete model description in [20].

Like most fishery age-structured models, the POP model has an essential simplicity at its core. Each cohort (represented as a diagonal of bubbles in Figure 6A) experiences successive depletion from annual removals by the fishery, analogous to the simple model in Table 1. Combining removals with the effects of selectivity and natural mortality gives the pattern in Figure 6B, where suitable recruitment parameters scale the cohorts sizes relative to each other. Despite this simplicity, however, a complete mathematical statement of the stochastic model and its likelihood functions occupies several pages dense with equations and notation [20, p. 23–29].

When scientists meet to debate the legitimacy of such a model, the discussion can become highly technical, often using terminology that alienates fishermen and other stakeholders. Furthermore, analysts typically invest substantial effort in writing computer code, running various analyses, producing tables, and crafting figures. All this hard work can generate resistance to new ideas, which might be difficult to include in an otherwise tidy framework. Like the legendary sculptor Pygmalion who fell in love with his own ivory statue, analysts can become attached to their models. Picture a cartoon scenario at LTCM in August, 1998: "Sir, now that Russia has defaulted on its bonds, should I trash the model that made 40% annually and start again?"

Skeptical fisherman James O'Malley sensed this problem in his claim [16] that "mathematics has been elevated to a status which suppresses knowledge and actually detracts from our efforts to acquire knowledge." In his view, the problem is "not mathematics per se, but the place of idolatry we have given it. ... Like any priesthood, it has developed its own language, rituals and mystical signs to maintain its status, and to keep a befuddled congregation subservient, convinced that criticism is blasphemy. Late at night, of course, many members of the scientific community will confess their doubts. But in the morning, they reappear to preach the catechism once again."

I can easily find late-night reasons for doubts about the POP model. It represents only one stock along the British Columbia coastline, chosen for the availability of historical survey data. The precise spatial definition of this stock is somewhat arbitrary, and its genetic extent remains uncertain. Although POP have a complex spatial distribution associated with geological features of the continental slope, the

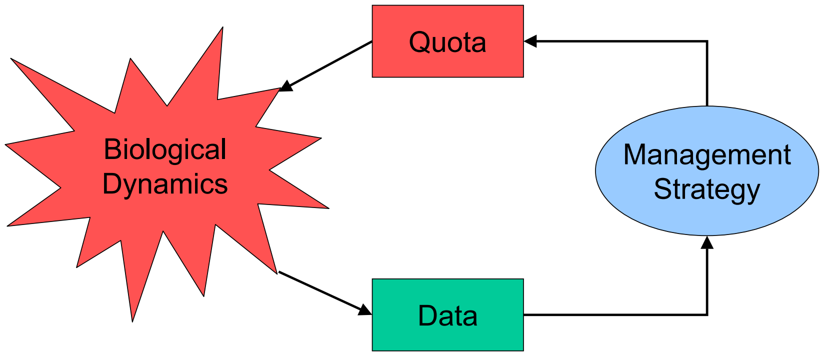

FIGURE 7. Robust management strategy. Uncertain biological dynamics generate observed data from which a management strategy, specified as a mathematical algorithm, determines the next allowable quota. This decision influences the dynamics during the next management cycle. The future quota remains unknown until calculated from the data. What algorithms define sensible management policies over a broad spectrum of potential dynamic systems?

model includes no spatial structure at all. Furthermore, the fishery that captures POP also removes significant amounts of about 100 other species, and smaller amounts of many others. Perhaps the most reassuring message comes directly from the catch data in Figure 5. After a history of much higher catches, modest removals have been sustained for more than two decades.

4. Decisions and uncertainty

In economics, *risk* is quantifiable, but *uncertainty* is not. For example, if a process can follow one of three scenarios A, B, or C, then risk analysis requires knowing the corresponding probability (p_A, p_B, p_C) for each scenario, where $p_A+p_B+p_C = 1$. Casinos and lotteries, with games that have known outcomes and probabilities, can exploit risk analysis to the fullest. Investors and fisheries managers rarely have the benefit of such clear understanding. Typically, they face uncertainty with unknown event probabilities and an incomplete list of potential scenarios. For example, future events might actually follow scenario X not contemplated when formulating policies to deal with A, B, or C.

Mathematics proceeds from assumptions to conclusions, and a mathematical model necessarily describes a chosen set of scenarios. The estimation model in Figure 3 limits data interpretation to a specified framework of dynamics and observation. In practice, the analyst usually looks at several alternative models to see how much the interpretation changes. But perhaps this entire approach is wrong. If, for example, the goal is to set a catch quota for the next year, then why worry about the profusion of estimates that spill out of a complicated analysis like the one discussed for POP? All we really need is a formula or algorithm to calculate the (currently unknown) future quota from the observed data.

Figure 7 portrays the decision problem from this point of view. We want a management strategy that is robust to an unknown biological system. For example, suppose that we seek to operate a fishery based on a time series of known catches C_t and biomass indices I_t. Then what function F should we use to set the catch quota

$$Q_{n+1} = F(C_1, \dots, C_n, I_1, \dots, I_n; \theta) \tag{4.1}$$

for time $t = n+1$, given data up to time $t = n$? This formulation allows a specified parameter vector θ to configure the rule for particular circumstances and policy objectives. For example, the recruitment age might be used to set time lags in the definition of F. Other components of θ might deal with risk tolerance, margins for error, and catch stability. When this policy is implemented in year $n+1$, the actual catch C_{n+1} normally equals the quota, but conceptually Figure 7 and formula (4.1) admit the possibility of *implementation error* with $C_{n+1} \neq Q_{n+1}$.

Fishery literature in recent years (e.g., [5] and [18]) has begun to investigate problems similar to the one posed in Figure 7. Unlike optimal control theory, this research seeks pragmatic controls that work sensibly across a spectrum of biological models, where the actual model remains unknown. For example, the policy (4.1) might simply be to stay the course:

$$Q_{n+1} = C_n\,, \tag{4.2}$$

as has happened recently in the herring and POP fisheries (Figures 2A and 5). Under what circumstances would this be a good or bad policy? Furthermore, could some really clever algorithm (4.1) guide fishery management toward a policy robust to nature's unpredictability? As illustrated in a different context by the Black-Scholes formula (2.1), a standard calculation based on a relatively simple model might give acceptable results most of the time.

Fishery scientists use simulation models with realistic biological complexity to evaluate management strategies like (4.1) and tune the control parameters θ to meet management objectives [18]. In practice, each new year n provides an opportunity to review and update the strategy in light of new information. This allows the process to be guided by human judgment and reduces the risk of disaster, but no model can reduce that risk to zero. Like financial markets altered by a crisis, nature sometimes changes the perceived "rules" governing fish population dynamics [21].

During the last few decades, the discovery of chaotic behavior has stimulated extensive research into nonlinear dynamic models. May [13] characterizes the spirit of this revolution in the 1970s by quoting a line from the play *Arcadia* by Tom Stoppard: "It's the best possible time to be alive, when almost everything you knew is wrong." The boundary between determinism and chance became blurred when scientists realized that deterministic models from an orderly Newtonian tradition could generate highly complex trajectories. Kuznetsov [10] illustrates the depth and variety of results that have emerged in this field. Brauer and Castillo-Chávez [2] incorporate some of these developments in their modern treatment of biological models. By discussing the historical and conceptual context for each model, they also guide readers toward appropriate model application.

While mathematicians developed the new science of chaos theory, statisticians began exploring new approaches to data analysis made possible by the computer revolution. For example, in 1979 Efron [6] speculated that computationally intensive methods might bypass the need for statistical theory. In their textbook written 14 years later, Efron and Tibshirani [7, p. xiv] explain that "The traditional road to statistical knowledge is blocked, for most, by a formidable wall of mathematics.

Our approach here avoids that wall. The bootstrap is a computer-based method of statistical inference that can answer many real statistical questions without formulas." This technique belongs properly to frequentist statistics, in which parameters have actual values in nature and their estimates inherit a distribution from the data. The bootstrap algorithm involves resampling the data with replacement and generating an empirical distribution of parameter estimates.

A similar computational revolution has also taken place in Bayesian statistics, which uses probability distributions to describe subjective uncertainty in parameter values. Starting from an initial *prior* distribution, new data determine a revised *posterior* distribution. The proposed algorithms use clever methods to generate a random sample of parameter values from the posterior, and this sample represents the analyst's current understanding. Clifford ([3], p. 53) emphasized the practical impact of one sampling method: "... from now on we can compare our data with the model that we actually want to use rather than a model which has some mathematically convenient form. This is surely a revolution".

To some extent, these historical developments have drawn mathematicians into two camps. Using deliberate oversimplification, I'll call them *theorists* and *realists*. In this hypothetical world, a theorist explores deterministic models to find interesting theorems and sometimes surprising dynamic behavior. Scenarios emerge from first principles. For example, different regions of parameter space might produce qualitatively different trajectories – stable, periodic, or chaotic. Data come into the discussion only after a clearly formulated understanding of how things might work. In fact, too much attention to data might restrict the theorist's range of imagination and exploration.

A realist, on the other hand, considers the data paramount, as in the quote from Sherlock Holmes [4]: "It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts." Data never fit a deterministic model exactly, so any analysis falls automatically into the realm of statistics. The focus shifts from theorems to algorithms, from abstract theory to detective work.

Given these stereotypes, it's easy to imagine areas of disagreement. The realist asks: "Why bother proving existence theorems about models that don't fit any data?" And the theorist counters: "How can you trust your complex model framework, with unknown global properties and limited computer explorations that blunder around randomly in the dark?" Perhaps like many readers of this paper, I consider myself an applied mathematician with roots in both camps. Applied work demands respect for reality, not just mathematical elegance (Samuelson [15]): "... don't believe in something because it's a beautiful formula. There will always be room for judgment." I'll give the final words to fisherman James O'Malley, who puts mathematics in a singular position between knowledge and understanding about the real world [16]:

"What is happening out there on the ocean, and why is it happening? What will we do about it? ... We owe it to ourselves, to the ocean, and especially to science itself, to assemble that great body of knowledge, those millions of observations, and to use every tool, including mathematics, to further our understanding of that knowledge. Knowledge and understanding are not the same. They may, in fact, be separated by a wide chasm. Mathematics is neither knowledge nor understanding. It may be a useful tool to help up bridge that gap. That is where it belongs, that is how we should use it, and we need to start now – before the bean-counters destroy us all."

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