

Constrained and generalized barycentric Davenport constants

*Las constantes baricéntricas generalizada
y restringida de Davenport*

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Abstract

Let G be a finite abelian group. The constrained barycentric Davenport constant $BD^s(G)$ with $s \geq 2$, is the smallest positive integer d such that every sequence with d terms in G contains a k -barycentric subsequence with $2 \leq k \leq s$. The generalized barycentric Davenport constant $BD_s(G)$, $s \geq 1$, is the least positive integer d such that in every sequence with d terms there exist s disjoint barycentric subsequences. For $s = 1$, this is just the barycentric Davenport constant $BD(G)$. Relations among $BD^s(G)$, $BD_s(G)$ and $BD(G)$ are established, these constants are related to the Davenport constant $D(G)$. Some values or bounds of $BD^s(G)$, $BD_s(G)$ and $BD(G)$ are given.

Key words and phrases: barycentric sequence; constrained barycentric Davenport constant; generalized barycentric Davenport constant; barycentric Davenport constant; Davenport constant; zero-sum.

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Resumen

Sea G un grupo abeliano finito. La constante restringida baricéntrica de Davenport $BD^s(G)$ con $s \geq 2$, es el más pequeño entero positivo d tal que toda sucesión con d términos en G contiene una subsucesión k -baricéntrica con $2 \leq k \leq s$. La constante generalizada baricéntrica de Davenport $BD_s(G)$, $s \geq 1$ es el más pequeño entero positivo d tal que toda sucesión con d términos, contiene s subsucesiones baricéntricas disjuntas, $s = 1$ corresponde a la denominada constante de Davenport baricéntrica $BD(G)$. Se establecen relaciones entre $BD^s(G)$, $BD_s(G)$ y $BD(G)$. Estas constantes están relacionadas con la constante $D(G)$ de Davenport. Damos algunos valores o cotas de $BD^s(G)$, $BD_s(G)$ y $BD(G)$.

Palabras y frases clave: sucesiones baricéntricas; constante restringida baricéntrica de Davenport; constante generalizada baricéntrica de Davenport; constante de Davenport baricéntrica; constante de Davenport; suma-cero.

1 Introduction

Let G be an abelian group of order n . The study of barycentric sequences starts in [6] and [8]. A sequence in G is barycentric if it contains one element which in the “average” of its terms. Formally, it is defined as follows:

Definition 1 ([8]). *Let A be a finite set with $|A| \geq 2$ and G an abelian group. A sequence $f : A \rightarrow G$ is barycentric if there exists $a \in A$ such that $\sum_A f = |A|f(a)$. The element $f(a)$ is called its barycenter.*

The word sequence is used to associate set A with set $\{1, 2, \dots, |A|\}$. That is to say $f = a_1, a_2, \dots, a_{|A|}$, where a_i are elements in G not necessarily distinct. When $|A| = k$ we shall speak of a k -barycentric sequence. Moreover when f is injective, the word *barycentric set* is used instead of barycentric sequence.

Notice that a sequence a_1, a_2, \dots, a_k is k -barycentric with barycenter a_j if and only if $a_1 + a_2 + \dots + (1 - k)a_j + \dots + a_k = 0$. So that a k -barycentric sequence is a weighted sequence with zero-sum. The investigations on sequences with zero-sum started in 1961 with a result of Erdős, Ginzburg and Ziv who proved that every sequence of length $2n - 1$ in G , contain an n -subsequence with zero-sum [11]. In 1966 [5] Davenport posed the problem to determine the smallest positive integer d , i.e. the Davenport constant $D(G)$, such that every sequence of length d contains a subsequence with zero-sum. The result

of Erdős, Ginzburg and Ziv and the introduction of the Davenport constant, gave origin to the zero-sum theory. This area has been given much attention lately, mostly because questions here occur naturally in other classic areas such as combinatorics, number theory and geometry. The state of the art on results, problems and conjectures on zero-sum theory and on barycentric theory is covered in the surveys [3], [14] and [22]. More information on zero-sum theory can be found in [1, 2, 4, 13, 15, 16, 17, 18, 19, 21] and on barycentric theory in [6, 8, 12, 20].

In [7] the generalized Davenport constant $D_s(G)$ and the constrained Davenport constant $D^s(G)$ are defined as $BD_s(G)$ and $BD^s(G)$, where the “barycentric subsequence” and “ k -barycentric subsequence” terms, are replaced by “subsequence with zero-sum” and “ k -subsequence with zero-sum” respectively. Also in [7], these constants provided new bounds for the Davenport constant for groups of rank three of type: $\mathbb{Z}_n \oplus \mathbb{Z}_{nm} \oplus \mathbb{Z}_{nmq}$ for $n = 2, 3$.

Definition 2 ([8]). *The barycentric Davenport constant $BD(G)$ is the least positive integer m such that every m -sequence in G contain barycentric subsequences.*

In what follows we consider p a primer number and \mathbb{Z}_p^s as a vector space on the field \mathbb{Z}_p . We will denote by $\{e_1, e_2, \dots, e_s\}$ the canonical basis of \mathbb{Z}_p^s , i.e. e_i , is the s -tuple with entry 1 at position i and 0 elsewhere.

Remark 1. *The vectorial subspaces of dimension 1 are defined as the lines of \mathbb{Z}_3^s . Moreover, a 3-subset in \mathbb{Z}_3^s is barycentric if and only if it is a line of \mathbb{Z}_3^s or equivalently its elements sum 0.*

We use the following theorems:

Theorem 1 ([9]). *In \mathbb{Z}_3^2 a set of 4 points is line-free if and only if it is a parallelogram. Moreover the maximum number of line-free points in \mathbb{Z}_3^2 is 4.*

For example, the parallelogram $(0, 0), (1, 0), (0, 1), (1, 1)$ is line-free.

Theorem 2 ([9]). *The maximum line-free set E in \mathbb{Z}_3^3 is 9.*

Moreover, a set of 9 points (x, y, z) is line-free if and only if they are distributed, up to affine isomorphism, as follows: $(2, 2, 2), (2, 1, 2), (1, 1, 2), (1, 2, 2), (0, 2, 1), (2, 0, 1), (1, 0, 1), (0, 1, 1), (0, 0, 0)$.

Theorem 3 ([9, 10]). *The maximum line-free set E in \mathbb{Z}_3^4 is 20.*

For example the set: $(0, 0, 0, 0), (2, 0, 0, 0), (0, 2, 0, 0), (2, 2, 0, 0), (1, 0, 2, 0), (0, 1, 2, 0), (1, 2, 2, 0), (2, 1, 2, 0), (1, 1, 1, 0), (1, 1, 0, 1), (0, 0, 2, 2), (2, 0, 2, 2),$

$(0, 2, 2, 2), (2, 2, 2, 2), (1, 0, 0, 2), (0, 1, 0, 2), (1, 2, 0, 2), (2, 1, 0, 2), (1, 1, 1, 2), (1, 1, 2, 1)$, is line-free.

The main goal of this paper is to introduce the constrained and generalized Davenport constants. Moreover relationships between these constants are established and some values or bounds are given.

2 Constrained barycentric Davenport constant

Definition 3. Let G be a finite abelian group, the constrained barycentric Davenport constant $BD^s(G)$ with $s \geq 2$ is the smallest positive integer d such that every sequence of length d in G , contains a k -barycentric subsequence with $2 \leq k \leq s$.

Remark 2. Let G be a finite abelian group then a sequence f contains a 2-barycentric sequence if and only if f has two equal elements.

In the following lemma the existence of $BD^s(G)$ is established.

Lemma 1. Let G be a finite abelian group. Then $BD^s(G) \leq |G| + 1$ for $s \geq 2$.

Proof. It is clear that in every sequence of length $|G| + 1$ there exist two equal elements. Therefore by Remark 2 we have a 2-barycentric sequence and then the lemma follows from definition of $BD^s(G)$. \square

The following theorem shows the relationship between $BD(G)$ and $BD^s(G)$.

Theorem 4. Let G be a finite abelian group of order n , then $BD(G) \leq BD^s(G)$. Moreover $BD(G) \leq n + 1$.

Proof. Trivially by definition of $BD(G)$ and Lemma 1. \square

Theorem 5. Let G be a finite abelian group of order n then $BD^2(G) = n + 1$.

Proof. By Lemma 1, we have $BD^2(G) \leq n + 1$. Moreover, by Remark 2, the set constituted by the n different elements of G , does not contain 2-barycentric sequences. So that $n + 1 \leq BD^2(G)$. \square

Theorem 6. Let G be a finite abelian group. If $BD(G) \leq s$ then $BD^s(G) = BD(G)$.

Proof. If $BD(G) \leq s$ then every sequence of length $BD(G)$ contains a t -barycentric sequence with $t \leq BD(G) \leq s$. Therefore by $BD^s(G)$ definition we have $BD^s(G) \leq BD(G)$. Moreover, by Theorem 4, we have $BD(G) \leq BD^s(G)$. Hence $BD^s(G) = BD(G)$. \square

Corollary 1. *Let G be a finite abelian group of order n then $BD^s(G) = BD(G)$, for $s \geq n + 1$.*

Proof. Directly from Theorem 4 and Theorem 6. □

We use the following two results.

Theorem 7 ([8, 23]). $BD(\mathbb{Z}_2^t) = t + 2$ for $t \geq 1$.

Theorem 8 ([8]). *For $t \geq 2$ we have $2t + 1 \leq BD(\mathbb{Z}_3^t) \leq 2t + 2$. Moreover $BD(\mathbb{Z}_3^t) = 2t + 1$, for $t = 1, 2, 3, 4, 5$.*

We have the following theorem and corollary:

Theorem 9. *Let G be a finite abelian group then $BD^{s+1}(G) \leq BD^s(G)$.*

Proof. Trivially by definition of $BD^s(G)$. □

Corollary 2. *Let G be a finite abelian group then $BD^s(G) \leq BD^3(G)$, for $s \geq 4$.*

Proof. Directly from Theorem 9. □

In the following two theorems some particular results of $BD^s(G)$ are given:

Theorem 10.

1. $BD^3(\mathbb{Z}_2^2) = 5$.
2. $BD^3(\mathbb{Z}_2^3) = 9$.
3. $BD^s(\mathbb{Z}_2^t) = t + 2$ for $s \geq t + 2$ and $t \geq 1$.

Proof.

1. By Theorem 9, $BD^3(\mathbb{Z}_2^2) = BD^{2+1}(\mathbb{Z}_2^2) \leq BD^2(\mathbb{Z}_2^2) = 5$. The sequence $(0, 0), (0, 1), (1, 0), (1, 1)$ does not contain k -barycentric subsequences with $k \leq 3$. Therefore $BD^3(\mathbb{Z}_2^2) \geq 5$, then $BD^3(\mathbb{Z}_2^2) = 5$.
2. By Theorem 9 we have $BD^3(\mathbb{Z}_2^3) = BD^{2+1}(\mathbb{Z}_2^3) \leq BD^2(\mathbb{Z}_2^3) = 9$, then $BD^3(\mathbb{Z}_2^3) \leq 9$. The sequence: $(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)$ does not contain k -barycentric subsequences with $k \leq 3$. Therefore $BD^3(\mathbb{Z}_2^3) = 9$.
3. By Theorem 7 we have $BD(\mathbb{Z}_2^t) = t + 2 \leq s$ with $t \geq 1$. By Theorem 6 we have $BD^s(\mathbb{Z}_2^t) = BD(\mathbb{Z}_2^t) = t + 2$ for $s \geq t + 2$.

□

Theorem 11.

1. $BD^3(\mathbb{Z}_3^2) = 5$.
2. $BD^3(\mathbb{Z}_3^3) = 10$.
3. $BD^s(\mathbb{Z}_3^t) = 2t + 1$ for $s \geq 2t + 1$ and $1 \leq t \leq 5$.

Proof.

1. Directly from Theorem 1
2. Directly from Theorem 2.
3. By Theorem 8 we have $BD(\mathbb{Z}_3^t) = 2t + 1 \leq s$ with $t = 1, 2, 3, 4, 5$, and by Theorem 6 we have $BD^s(\mathbb{Z}_3^t) = BD(\mathbb{Z}_3^t) = 2t + 1$ for $s \geq 2t + 1$ and $1 \leq t \leq 5$.

□

3 Generalized barycentric Davenport constant

Definition 4. Let G be a finite abelian group. The generalized barycentric Davenport constant $BD_s(G)$ with $s \geq 1$, is the least positive integer d such that every sequence of length d in G , contains s disjoint barycentric subsequences.

It is clear that $BD_1(G) = BD(G)$.

The following lemma proves the existence of $BD_s(G)$.

Lemma 2. Let G be a finite abelian group. Then $BD_s(G) \leq sBD(G)$.

Proof. It is clear that every sequence f of length $sBD(G)$ can be partitioned in s disjoint subsequences of length $BD(G)$. □

Lemma 3. Let G be a finite abelian group. Then $BD_s(G) \leq BD_{s+1}(G)$.

Proof. Trivially by definition of $BD_s(G)$. □

The following lemma shows the relationship between $BD_s(G)$ and $BD^s(G)$.

Lemma 4. Let G be a finite abelian group. If $BD^s(G) \leq BD_i(G) + s$ then $BD_{i+1}(G) \leq BD_i(G) + s$.

Proof. Let f be a sequence of length $BD_i(G) + s$. Since $BD^s(G) \leq BD_i(G) + s$ then there exists a t -barycentric sequence with $t \leq s$. Therefore from the remaining $BD_i(G)$ terms of f , i disjoint barycentric sequence can be formed. Hence we have the lemma. □

Corollary 3. *Let G be a finite abelian group. If $BD^s(G) \leq BD_i(G) + s$ then $BD_{i+n}(G) \leq BD_i(G) + ns$.*

Proof. Directly from Lemma 4. □

We have the following result:

Theorem 12. $BD_n(\mathbb{Z}_2) = BD_n(\mathbb{Z}_3) = BD_n(\mathbb{Z}_4) = 2n + 1$ for $n \geq 1$.

Proof.

- Since $BD_1(\mathbb{Z}_2) = 3$ and $BD^2(\mathbb{Z}_2) = 3$ then $BD^2(\mathbb{Z}_2) = 3 < 5 = BD_1(\mathbb{Z}_2) + 2$. Therefore by Corollary 3 we have $BD_{1+(n-1)}(\mathbb{Z}_2) \leq BD_1(\mathbb{Z}_2) + (n-1)2 = 3 + 2n - 2 = 2n + 1$. Hence we obtain $BD_n(\mathbb{Z}_2) \leq 2n + 1$.
- Since $BD_1(\mathbb{Z}_3) = 3$ and $BD^2(\mathbb{Z}_3) = 4$ then $BD^2(\mathbb{Z}_3) = 4 < 5 = BD_1(\mathbb{Z}_3) + 2$. Therefore by Corollary 3 we have $BD_{1+(n-1)}(\mathbb{Z}_3) \leq BD_1(\mathbb{Z}_3) + (n-1)2 = 3 + 2n - 2 = 2n + 1$. Hence we obtain $BD_n(\mathbb{Z}_3) \leq 2n + 1$.
- Since $BD_1(\mathbb{Z}_4) = 3$ (see [8]) and $BD^2(\mathbb{Z}_4) = 5$ then $BD^2(\mathbb{Z}_4) = 5 = BD_1(\mathbb{Z}_4) + 2$. Therefore by Corollary 3 we have $BD_{1+(n-1)}(\mathbb{Z}_4) \leq BD_1(\mathbb{Z}_4) + (n-1)2 = 3 + 2n - 2 = 2n + 1$. Hence we obtain $BD_n(\mathbb{Z}_4) \leq 2n + 1$.
- The sequence f of length $2n$ constituted by 1 and $2n-1$ zeros, considered as a sequence of \mathbb{Z}_2 or \mathbb{Z}_3 or \mathbb{Z}_4 , contains at most $n-1$ 2-barycentric sequences. Therefore $2n+1 \leq BD_n(\mathbb{Z}_2)$, $2n+1 \leq BD_n(\mathbb{Z}_3)$ and $2n+1 \leq BD_n(\mathbb{Z}_4)$.

Consequently the theorem is proved. □

Theorem 13. $BD_n(\mathbb{Z}_2^2) = 2n + 2$ for $n \geq 1$.

Proof. By Theorem 7, we have $BD_1(\mathbb{Z}_2^2) = 4$ and by Theorem 5, we have $BD^2(\mathbb{Z}_2^2) = 5$. Therefore $BD^2(\mathbb{Z}_2^2) = 5 < 6 = BD_1(\mathbb{Z}_2^2) + 2$. Hence, by Corollary 3, we have: $BD_{1+(n-1)}(\mathbb{Z}_2^2) \leq BD_1(\mathbb{Z}_2^2) + 2(n-1)$, i.e., $BD_n(\mathbb{Z}_2^2) \leq 2n + 2$. On the other hand, the sequence $f = e_1, e_2, e_1 + e_2, 0, 0, \dots, 0$ with $2n-2$ zeros, has length $2n+1$. The first 4 elements constitute a 4-barycentric sequence with 0 as barycenter. Moreover from the remaining $2n-3$ zeros, we can form $n-2$ disjoint 2-barycentric subsequences. Therefore f contains at most $n-1$ disjoint barycentric subsequences. Hence $BD_n(\mathbb{Z}_2^2) \geq 2n + 2$. Consequently, $BD_n(\mathbb{Z}_2^2) = 2n + 2$. □

Theorem 14. $BD_n(\mathbb{Z}_2^3) = 2n + 4$, for $n \geq 2$.

Proof. Let f be a sequence of length 8 in \mathbb{Z}_2^3 . If f is injective then $f = \{0, e_1, e_2, e_3, e_1+e_2, e_1+e_3, e_2+e_3, e_1+e_2+e_3\}$. Therefore we have the following two disjoint barycentric sequence: $0, e_1, e_2, e_3, e_1 + e_2$ and $e_1 + e_3, e_2 + e_3, e_1 + e_2 + e_3$.

If sequence f is not injective, then we have a 2-barycentric sequence and from the other 6 remaining element of f , we obtain the second disjoint barycentric sequence. Therefore $BD_2(\mathbb{Z}_2^3) \leq 8$. Let f be the sequence of length $2n + 3$ in \mathbb{Z}_2^3 constituted by 8 different elements and one of them repeated $2n - 4$ times. Since $BD_2(\mathbb{Z}_2^3) \leq 8$ then from the 8 different elements we obtain two disjoint barycentric sequences and from the remaining $2n - 5$ elements, we obtain $n - 3$ disjoint 2-barycentric sequences. Therefore in f we can identify at most $n - 1$ disjoint barycentric sequences, i.e. $BD_n(\mathbb{Z}_2^3) \geq 2n + 4$.

In particular $BD_2(\mathbb{Z}_2^3) \geq 8$, so that $BD_2(\mathbb{Z}_2^3) = 8$. By Theorem 5, $BD^2(\mathbb{Z}_2^3) = 9$. Hence $BD^2(\mathbb{Z}_2^3) = 9 < 8 + 2 = BD_2(\mathbb{Z}_2^3) + 2$. So that $BD^2(\mathbb{Z}_2^3) \leq BD_2(\mathbb{Z}_2^3) + 2$. Applying Corollary 3, we have $BD_n(\mathbb{Z}_2^3) = BD_{2+(n-2)}(\mathbb{Z}_2^3) \leq BD_2(\mathbb{Z}_2^3) + 2(n - 2) = 8 + 2n - 4 = 2n + 4$. Consequently, $BD_n(\mathbb{Z}_2^3) = 2n + 4$. \square

Theorem 15. $BD_n(\mathbb{Z}_3^2) = 2n + 3$, for $n \geq 2$.

Proof. Let f be a finite sequence with 7 elements in \mathbb{Z}_3^2 . If f is not injective we have a 2-barycentric sequence and since $BD(\mathbb{Z}_3^2) = 5$ (Theorem 8) then from the 5 remaining elements in f we obtain another barycentric sequence. So that in f we can identify 2 disjoint barycentric sequences. Assume that f is injective, then by Theorem 1 in each 5 elements there is a 3-barycentric sequence i.e. a line L in \mathbb{Z}_3^2 . If, in the remaining 4 elements there is a barycentric sequence, hence we obtain two disjoint barycentric sequences. Otherwise, by Theorem 1, these 4 elements form a parallelogram. By a simple inspection, there exist two parallel lines, using two different points p, q of the parallelogram, intersecting line L in two different points and using the other two points of the parallelogram. Then, we obtain two disjoint 3-barycentric sets. Therefore $BD_2(\mathbb{Z}_3^2) \leq 7$. On the other hand, the sequence $0, e_1, e_2, 2e_1, 2e_2, e_1 + e_2$ does not contain two disjoint barycentric sequences. Hence $BD_2(\mathbb{Z}_3^2) \geq 7$. So that:

$$BD_2(\mathbb{Z}_3^2) = 7 = 2 \cdot 2 + 3. \quad (1)$$

Let f be a sequence in \mathbb{Z}_3^2 of length 9. If f is not injective then we have a 2-barycentric sequence. From the remaining 7 elements, there exist, by equation 1, two disjoint barycentric sequences. Hence in f there are three

disjoint barycentric sequences. If f is injective i.e. $f = \mathbb{Z}_3^2$, we have the following three disjoint 3-barycentric sets: $\{0, e_1, 2e_1\}$, $\{e_2, e_1 + e_2, 2e_1 + e_2\}$, $\{2e_2, e_1 + 2e_2, 2e_1 + 2e_2\}$. Therefore $BD_3(\mathbb{Z}_3^2) \leq 9$.

On the other hand, the sequence $0, e_1, e_2, 2e_1, e_1 + e_2, e_1 + 2e_2, 2e_1 + 2e_2, 2e_1 + e_2$ does not contain three disjoint barycentric sets. So that $BD_3(\mathbb{Z}_3^2) \geq 9$. Hence $BD_3(\mathbb{Z}_3^2) = 9$. By Theorem 5 $BD^2(\mathbb{Z}_3^2) = 10 < 11 = 9 + 2 = BD_3(\mathbb{Z}_3^2) + 2$. By Corollary 3, we have $BD_n(\mathbb{Z}_3^2) = BD_{3+(n-3)}(\mathbb{Z}_3^2) \leq BD_3(\mathbb{Z}_3^2) + 2(n-3) = 9 + 2n - 6 = 2n + 3$, i.e. $BD_n(\mathbb{Z}_3^2) \leq 2n + 3$.

Moreover, the sequence in \mathbb{Z}_3^2 of length $2n+2$ with 9 different elements and one of them repeated $2n-6$ times, contains at most $n-1$ disjoint barycentric sequences. Hence $BD_n(\mathbb{Z}_3^2) \geq 2n + 3$. Consequently $BD_n(\mathbb{Z}_3^2) = 2n + 3$. \square

Theorem 16. $BD_n(\mathbb{Z}_3^3) \leq 3n + 4$, for $n \geq 2$ and $BD_2(\mathbb{Z}_3^3) = 10$.

Proof. Let f be a finite sequence with 10 elements in \mathbb{Z}_3^3 . If f is not injective we have a 2-barycentric sequence and since $BD(\mathbb{Z}_3^3) = 7$ then from the remaining elements in f we obtain an other barycentric sequence. Assume that f is injective, then by Theorem 2 in each 10 different elements there is a 3-barycentric set; the second one is obtained from the remaining 7 elements. So that $BD_2(\mathbb{Z}_3^3) \leq 10$. Moreover since in the 9 elements $(2, 2, 2), (2, 1, 2), (1, 1, 2), (1, 2, 2), (0, 2, 1), (2, 0, 1), (1, 0, 1), (0, 1, 1), (0, 0, 0)$, there are not no two disjoint barycentric sets, we have $BD_2(\mathbb{Z}_3^3) = 10$. On the other hand, since $BD^3(\mathbb{Z}_3^3) = 10$ we have: $BD^3(\mathbb{Z}_3^3) = 10 < BD_2(\mathbb{Z}_3^3) + 3$. Hence by Corollary 3, we have $BD_n(\mathbb{Z}_3^3) = BD_{2+(n-2)}(\mathbb{Z}_3^3) \leq BD_2(\mathbb{Z}_3^3) + (n-2)3$. Therefore we obtain: $BD_n(\mathbb{Z}_3^3) \leq 10 + 3n - 6 = 3n + 4$. \square

Problem 1. Using Theorem 3, determine the exact value or bound of $BD_n(\mathbb{Z}_3^4)$ for $n \geq 2$.

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