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LARS-ERIK PERSSON–THE MAN AND HIS WORK

LECH MALIGRANDA[1]

Dedicated to Professor Lars-Erik Persson on the occasion of his 65th birthday

Communicated by M. S. Moslehian

ABSTRACT. On the occasion of Lars-Erik Persson's 65th birthday there was organized a conference *Analysis, Inequalities and Homogenization Theory (AIHT)*, which was held at Luleå University of Technology in June 8-11, 2009, in Luleå-Sweden. The first plenary lecture had the title above and here I present an abbreviated form of this lecture.

1. INTRODUCTION

The Department of Mathematics of Luleå University of Technology in Sweden and Narvik University in Norway organized a conference in June 2009 on the occasion of the 65th birthday of professor Lars-Erik Persson (LEP) entitled *Analysis, Inequalities and Homogenization Theory (AIHT) – midnightsun conference in honor of Professor Lars-Erik Persson on the occasion of his 65th birthday*, June 8-11, 2009, Luleå, Sweden. The main organizer was professor Lech Maligranda, who gave the first plenary lecture with the above mentioned title. The main intention of that lecture was to give some selected and personal information about LEP, his scientific achievements in mathematics and information about his cooperation with others, including also some personal reflections and the collection of all his 16 published books and 10 most cited papers. In this paper an abbreviated form of this lecture is presented.

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2. Some background data about LEP

Lars-Erik Persson was born on 24 September 1944 in Dorotea but the family was living in Svanabyn. Svanabyn is a village 30 km from Dorotea with 200 inhabitants. He finished here classes 1–6 and he was the first person from Svanabyn who went for further studies in Dorotea.

His parents Erik Arvid Persson (7 June 1914 – 1999) and Dagmar Sigrid Cecilia Persson (16 March 1919 – 19 July 2008), born Johansson, had 9 children but only 3 were really born and 1 was living 5 months and 1 only 5 days. Lars-Erik was number 8 on this list. What was the problem? The blood conflict. The doctor in Dorotea – Sten Strömbom (1886-1969) – understood this conflict. He took care about Lars-Erik and saved him. He was also very important person in his life in the future, for example he could stay in his home almost as adopted son during his studies at "realskolan" in Dorotea (corresponding to classes 7-9).

Photo 1. LEP 1 year old

The years 1944-1957 Lars-Erik was living in Svanabyn and next 4 years in Dorotea. In the years 1961-1964 he studied at the gymnasium in Östersund in the middle of Sweden. After that, in 1964, he did his military service in Umeå. The next five years (1965-1969) he studied mathematics, physics and other related subjects at Umeå University. Meanwhile in 1967 he married Annika Strömquist. They got three children born in 1967, 1969 and 1975.

In 1970 he finished also the teacher education at Umeå University and was even working half a year at a secondary school as a part of this education. In 1972 he obtained *Licenciate* degree (something like half of PhD) at Umeå University with a thesis entitled *Integrability conditions on periodic functions and their Fourier transforms* and in 1974 PhD degree with a thesis entitled *Relations between regularity of periodic functions and their Fourier series*: His supervisor was professor Ingemar Wik, the former student of the famous professor Lennart Carleson.

In the academic year 1974/75 he had a temporary position as associate professor and was teaching at Umeå University and from 1975 he became associate professor at Luleå University of Technology.

In 1987 he became *docent* (habilitation in other countries) at Luleå University of Technology, after scientific evaluation by Professor Jaak Peetre (Stockholm,

Photo 2. 1974. LEP with his parents after PhD defence

Lund) and Professor Gunnar Aronsson (Linköping). For 4 years between 1988 and 1993 he was acting professor in mathematics at Luleå University of Technology and in 1991 he married Lena Nilsson. They got two children born in 1991 and 1993. In the years 1992-1994 he was professor at Narvik University in Norway and from 1994 till now he is full professor at Luleå University of Technology. Moreover, from January 2004 he became also a professor at Uppsala University and is working there for 30 % of all his duties. At the moment he is also honorary professor at Eurasian National University in Kazakhstan and professor II at Narvik University, Norway.

3. LEP's two families

Let us say a little more about LEP's two families. He was married twice, has 5 children, 3 granddaughters and 1 grandson. More precisely, in August 1967 he married Annika Strömquist and they had three children: Margareta <u>Susanna</u> (born 1967; she has 2 daughters), Lars <u>Torbjörn</u> (born 1969; he has 1 son) and Erik <u>Zakarias</u> (born 1975; he has 1 daughter). In 1990 they divorced. On August 17, 1991 he married Lena Nilsson (born on June 3, 1962) and they have got 2 children: <u>Elin</u> Magdalena (born 1991) and <u>Hans</u> Erik (born 1993).

Photo 3. 17 August 1991. LEP's two families on the occasion of his wedding with Lena

4. Some information from LEP's CV

LEP was vice President (1994-1996) and then President (1996-1998) of the Swedish Mathematical Society. In the period 1995-2002 he has been the secretary of the Swedish National Committee for Mathematics at the Royal Academy of Sciences and he is still member of this committee. In 1995 he founded Centre of Applied Mathematics at Luleå University of Technology and in 2008 he became director of the Center of Interdisciplinary Mathematics (CIM) at Uppsala University.

Lars-Erik is member of the Editor Board of 8 international journals: *Journal of Function Spaces and Applications* (JFSA) (Editor-in-chief), *Mathematical Inequalities & Applications* (MIA), *Advances in Nonlinear Variational Inequalities*, *Banach Journal of Mathematical Analysis* (BJMA), *Composites: Part B Engineering journal*, *Journal of Inequalities in Pure and Applied Mathematics*, *Asian-European Journal of Mathematics*, *The Australian Journal for Mathematical Analysis and Applications*

I would also like to mention his important activities in the Swedish Research Council (Vetenskapsrådet=VR). Firstly, he has himself been supported for more then 15 years with research grants from VR and this is very rare in Sweden. Secondly, in 2008 he was elected as one of ten members in the board at VR, which decide about these research money (at the moment around 65 millions of Swedish crowns a year are decided by this board) in the subject *Natural and Engineering Sciences* group: *Mathematics and Technical Mathematics.* Moreover, in 2009 he was appointed as chairman of the same Committee.

Let us also say some words about LEP's contributions to make mathematics more popular in the Swedish schools. During the last years LEP has also been used in several ways as expert in mathematics to try to change the really bad situation for mathematics interest among students in the basic Swedish school system. He has given a great numbers of so called "inspiration lectures" both for teachers and students in the basic school system and even with people from the Swedish government as listeners. He has written a great number of articles in newspapers about that and also been in the national television in a debate e.g. with two representatives from the Swedish government, which decides over all education money in Sweden.

LEP has organized five conferences and he has given more than 40 talks at international conferences (most of these talks have been given as invited or plenary speaker).

Photo 4. 2001. LEP at the conference in Wrocław, Poland.
LEP is the second one from the left in the first row

Lars-Erik was/is supervisor of 54 (41 already finished including 17 females) Ph.D. students, mainly in mathematics:

1992 Nils Svanstedt; 1994 Thomas Strömberg; 1996 Andrejs Dunkels (mathematical didactics), Anders Holmbom, Dag Lukkassen; 1997 Stefan Ericsson; 1998 Niklas Wellander, Marianna Euler, Peter Wall; 2000 Sorina Barza; 2001 Ove Lindblom, Annette Meidell; 2002 Johan Byström, Maria Nassyrova; 2003 Niklas Grip, Leon Simula, Dmitry Prokhorov, Leo Larsson; 2004 Anna Wedestig; 2005 Evgueny Kuznetsov, Jonas Engström, Narsis Mtega; 2006 Aigerim Kalybay, Christopher Okpoti, Elena Ushakova, Monica Johansson (mathematical didactics), Kristina Juter (mathematical didactics), Örjan Hansson (mathematical didactics); 2007 Komil Kuliev, Gulchehra Kulieva, Alice Lesser; 2008 Maria Johansson, Emmanuel Kwame Essel; 2009 Zamira Abdikalikova, Ainur Temirkhanova, Lyazzat Sarybekova, Guy Beeri, Irina Pankratova; 2010 Per-Eskil Persson (mathematical didactics), Anca-Nicoleta Marcoci, Liviu-Gabriel Marcoci.

The work of LEP has been recognized in several ways. In particular, it has been written a lot of him in Swedish newspapers and he has been in Swedish radio and television several times. Moreover, he has got several prizes, awards and distinctions. Among others, I want to mention that he obtained in 2002 the first *"Louise Petréns" award* for his special support to female mathematicians in Sweden and the next year the *Orlicz Medal* for his contributions in mathematical research. In 2004 he received *Sammy Lindmark's award* (50.000 SEK) for his research and cooperation with the industry in northern Sweden and the next year the *Luleå University of Technology award* (40.000 SEK) for his outstanding cooperation with the world outside the university. In 2008 he received the prestigious *"Ångpanneföreningen" award* (100.000 SEK) for his outstanding work to transfer important knowledge to the world outside the university. This award is given each year for at most one researcher from all subjects and all universities in Sweden! LEP is the only mathematician who has got this award. In 2008 he received the Luleå University of Technology award (10.000 SEK) as the best supervisor at this university. This was the first time when this award was given to someone. Moreover, in 2005 he was appointed as honorary professor at Eurasian National University in Astana, Kazakhstan.

5. LEP in competitions

All activities of LEP look like a competition. Let us collect some of his achievements:

- As a young person he was playing football in the Svanabyn team and he was making several gools. Once even 4 gools!
- He has finished 50 times "Vasaloppet", that is, the famous Swedish 90 km skiing competition. His best time from 1983 was 4.56.
- He has finished 19 full marathon runnings (42 km); his best time 2.49 is from the Stockholm Marathon.
- The number of published papers in MathSciNet: 168 (227 at his Website).
- He has written joint papers/books with 144 persons from 37 countries [in this 86 publications with females from 15 countries].
- Number of citations in MathSciNet: 351 times by 248 authors.
- His *h-index* (Hirsch index) is 10.[1]
- His Erdös number is 3 (via Pečarić, Klamkin to Erdös). His Banach number is 4 and Lions number is 1.

[1]The *h-index* is an index that attempts to measure both the scientific productivity and the apparent scientific impact of a scientist. The index is based on the set of the scientist's most cited papers and the number of citations that they have received in other people's publications. The index was suggested by Jorge E. Hirsch in 2005.

- From 22 July 2003 he is a member of *"MathR 100–CLUB"*, that is, in the period 1973-2003 he published a lot of papers and some books, and from this time 100 were reviewed in *Mathematical Reviews*.
- He is at the moment appointed as professor in some sense at four universities (Luleå University of Technology, Uppsala University, Narvik University and Eurasian National University in Kazakhstan).
- He is the only one in north Sweden, who has been elected as President of the Swedish Mathematical Society and as Chairman in the Committee of Mathematics at the Swedish Research Council.
- He is the only Swedish mathematician who has been ordinary member of the Swedish National Committee of Mathematics at the Swedish Royal Academy of Sciences during all of the period 1995-still (during 1995-2002 he was even the secretary).

6. Some data about our cooperation

We met for the first time in August 1983 in Lund at the conference *Interpolation Spaces and Allied Topics in Analysis*, Aug. 29–Sept. 1, 1983, Lund, Sweden.

Photo 5. 1983. Conference in Lund, Sweden. Sitting from the left: Gunnar Sparr, Per Nilsson, Hrushikesh Narhar Mhaskar, Mario Milman, Sten Kaijser, Svante Janson. Standing from the left: Jan Gustavsson, Tord Sjödin, Erik Svensson, Jöran Bergh, Zeev Ditzian, Jörgen Löfström, Lars-Erik Persson, Jonathan Arazy, Boris Mityagin, William C. Connett, Jaak Peetre, Lech Maligranda, Claude Merucci and Jan Boman

Then, in 1986, I was in Lund and Luleå for 14 days in June. First time he visited me in Poznań, Poland, it was in 1986. After that, in 1987, he came to me together with his wife Annika and son Zakarias. He was very happy to learn two Polish words: *"Dziękuję"* (thank you) and *"Dzień Dobry"* (good morning) and was shocked with the Polish prices, e.g. of vodka, informing me that he was feeling in Poland with the Swedish salary – for the first time in his life – that he was a *"very rich man"*.

In 1987 I went to Venezuela (I was working there in the years 1987-1991). In June 1988 I visited LEP in Luleå for 19 days and in 1989 we both participated in the conference *Function Spaces II*, Poznań, Poland (Aug. 28–Sept. 2) and the enclosed photo with Orlicz is from this stay.

Photo 6. 1989. Poznań, Poland. From the left: Lars-Erik Persson, Władysław Orlicz, Wanda Matuszewska and Lech Maligranda

In September-October 1989 I was again 24 days with LEP in Luleå and in November-December 1989 he came to me to Caracas. In particular, I remember that he enjoyed very much to pick up himself bananas from my garden.

Our cooperation was important for both of us and, therefore, I was again visiting LEP in Luleå for 48 days in September–October 1990 and he came to me, for the second time, to Caracas in November 1990. In particular, he was referee for my PhD student Alexi Quevedo, who defended his thesis *Interpolation of some classes of operators* at the University Central of Venezuela (UCV) in Caracas on 27 November 1990.

From November 1991 I started my work at Luleå University of Technology. As colleagues from the same university we went together to five conferences in Poland called *Function Spaces III, IV, V, VI, VII* (Poznań, Aug. 31–Sept. 4, 1992; Zielona Góra, Aug. 28–Sept. 1, 1995; Poznań, Aug. 28–Sept. 3, 1998; Wrocław, Sept. 3–8, 2001 and Poznań, July 21–25, 2003), including also *The*

Władysław Orlicz Centenary Conference and Function Spaces VII on which we both got the *Orlicz Medal* after presenting our invited lectures.

As a friend and admirer of LEP, I organized the conference described in this paper and to the honor of LEP on the occasion of the 65th birthday of him. On the photo below we can see almost all the participants of the conference. They were 91 participants from 24 countries. A lot of information and complementary photos from the conference (e.g. when the participants took the important step over the Arctic Circle) can be found at the Website: `http://www.math.ltu.se/aiht/`.

Photo 7. 2009. Jokkmokk, Sweden. Participants of the AIHT conference with diplomas confirming their cross of the Arctic Circle

When I was preparing my lecture for this conference I asked LEP in May 2009: *What is mathematics for you?* He answered immediately:

> *Mathematics is the most fantastic subject which ever was created by human beings and also a subject which will survive all perturbations in popularity of all other subjects.*

Our scientific cooperation in the period 1986-2009 consist of a lot of activities including 2 joint published books and 27 joint published papers.

7. Some selected mathematical results of LEP

LEP's areas in mathematics, in historical order, are: Fourier Analysis (later on more general Harmonic Analysis), Interpolation Theory, Function Spaces, Homogenization Theory and Inequalities.

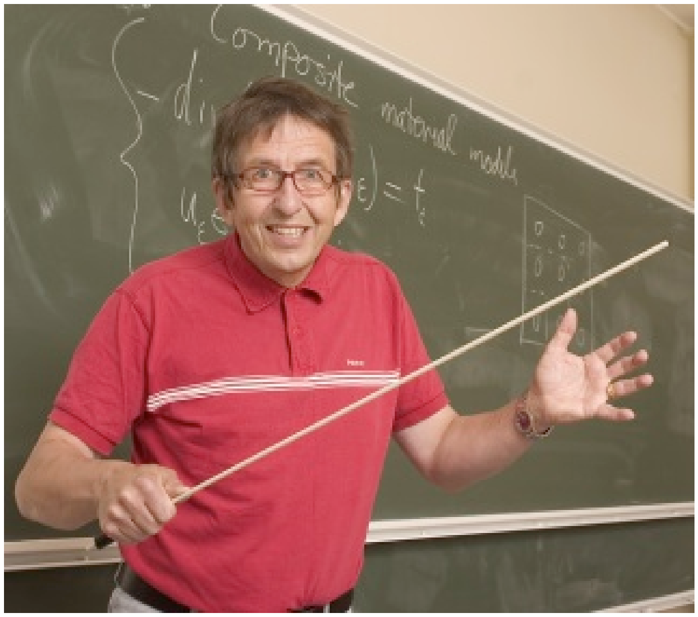

Photo 8. 2008. LEP enjoying mathematics

LEP has been author/coauthor of more than 200 papers and 15 books. It is of course impossible to try to give a fair description of his contributions so I only present a selection of results which are more closely related to his name and which I know best.

The first result, which has his name *Persson's formula* was proved in 1984:

$$\|x\|_{(L_{p_0}(A_0,w_0),L_{p_1}(A_1,w_1))_{\theta,q}} \approx \sup \|x\|_{L_1(A_0,A_1)_{g(t),1;L}} \text{ for } q > p_\theta. \tag{7.1}$$

We continued this off-diagonal description of the real K-method in the paper L. Maligranda and L.-E. Persson, *Real interpolation of weighted L^p- and Lorentz spaces*, Bull. Polish Acad. Sci. Math. 35(1987), 765–778.

This was, in fact, our first joint paper from all of 27 joint published papers and 2 books.

In our second joint paper [P2] we investigated the following multiplier space

$$M(X,Y) = \{m \in L^0 : mx \in Y \text{ for all } x \in X\},$$

which for $Y = L^1(\mu)$ becomes the classical Köthe dual space of X. If X, Y are Banach function spaces, then the functional $\|m\| = \sup\{\|mx\|\colon \|x\| \leq 1\}$ turns $M(X,Y)$ into a Banach function space. We found an exact description of these generalized dual spaces for such classical spaces as the X^p-spaces of Lozanovskii, Lorentz, Marcinkiewicz and Orlicz spaces.

As the best of our joint papers I count [P4], where we proved results on the weighted maximal function. For a fixed weight function (locally integrable positive function) w and any measurable function f on the n-dimensional Euclidean space $\mathbb{R}^n$ one defines the *maximal function* M_w by

$$M_w f(x) = \sup_{x \in Q} \frac{1}{w(Q)} \int_Q |f(y)|\, w(y)\, dy,$$

where the supremum is taken over all cubes Q, which contain x with sides parallel to the cordinate axes. If $w = 1$ we have classical maximal function $Mf(x)$. Four classical inequalities for the maximal function M are known:

- *the Riesz inequality*: $(Mf)^*(t) \le Af^{**}(t)$ for all $t > 0$,
- *the Wiener inequality*:

$$|\{x \in \mathbb{R}^n : \ Mf(x) > \lambda\}| \le \frac{B}{\lambda} \int_{\{x\in\mathbb{R}^n : |f(x)|>\lambda/2\}} |f(x)|\, dx \text{ for all } \lambda > 0,$$

- *the Stein inequality*:

$$\int_{\{x\in\mathbb{R}^n : |f(x)|>\lambda\}} |f(x)|\, dx \le C\lambda |\{x \in \mathbb{R}^n; Mf(x) > \lambda\}| \text{ for all } \lambda > 0,$$

- *the Herz inequality*: $f^{**}(t) \le D(Mf)^*(t)$ for all $t > 0$.

In the paper [P4] all these estimates were investigated in the weighted case and without doubling condition. The results are the following (Asekritova-Krugljak-Maligranda-Persson, 1997):

> *The Riesz inequality is equivalent to the Wiener inequality and they are true if and only if the maximal operator M_w is of weak type $(1, 1)$. The Stein inequality and the Herz inequality are valid without any restriction on the weight w.*

Moreover, the K-functional for the pair $(L^1(w), L^\infty)$ was calculated.

It should also be mentioned that if either $n = 1$ or $n \ge 2$ and $w \in D_2$ (doubling property), i.e. there exists a constant $C > 0$ such that $w(2Q) \le Cw(Q)$ for any cube Q, where $w(Q) = \int_Q w(t)\, dt$, then M_w is of weak type $(1, 1)$, that is,

$$\lambda\, w(\{x \in \mathbb{R}^n : M_w f(x) > \lambda\}) \le D \int_{\mathbb{R}^n} |f(x)|\, w(x)\, dx \text{ for all } \lambda > 0.$$

On the other hand, in $\mathbb{R}^2$ for the Gaussian measure $w(x, y) = e^{-x^2-y^2}$ and the Laplace measure $w(x, y) = e^{x+y}$ the maximal function M_w is not of weak type $(1, 1)$.

Interpolation of subspaces is an important and rather complicated problem in interpolation theory. A particular case of this general subspace interpolation problem is interpolation of intersections by the real method. This problem was investigated in paper [P6].

The problem of interpolation of intersections is the problem of finding conditions on a triple (X_0, X_1, N) and the parameters $\theta \in (0, 1)$ and $q \in [1, \infty]$ of the real interpolation method under which the natural equality formula (with equivalence of norms)

$$(X_0 \cap N, X_1 \cap N)_{\theta,q} = (X_0, X_1)_{\theta,q} \cap N$$

is valid. Here (X_0, X_1) is a Banach couple in a Hausdorff topological vector space X and N is a linear subspace of X, which generates a normed (in general, non-Banach) couple $(X_0 \cap N, X_1 \cap N)$, where the norm on $X_i \cap N$ is just obtained by the restriction of the norm of X_i $(i = 0, 1)$. When X_0, X_1 and N are all normed lattices on the same measure space, then such equality is valid and it was proved by Maligranda in 1985. Note that when N is the kernel of a linear functional $\psi \in (X_0 \cap X_1)^*$ this equality means just the equivalence of the norms of the spaces $N_{\theta,q} = (X_0 \cap N, X_1 \cap N)_{\theta,q}$ and $X_{\theta,q} = (X_0, X_1)_{\theta,q}$ on the subspace N. An important particular case of the problem on the interpolation of intersections generated by a linear functional was treated in the paper [P6] by Krugljak, Maligranda and Persson, where sufficient conditions on $\theta \in (0,1)$, $p \in [1,\infty)$ and on the weight functions w_0, w_1 were found ensuring that the relation

$$(L^p(w_0) \cap N, L^p(w_1) \cap N)_{\theta,p} = (L^p(w_0), L^p(w_1))_{\theta,p} \cap N$$

holds, where $L^p(w)$ is the usual weighted L^p-space on $(0,\infty)$ with the usual norm and N is the space of all functions $f\colon (0,\infty) \to \mathbb{R}$ such that $\int_0^\infty f(x)\,dx = 0$. Namely, the K-functional of Peetre is calculated for the couple $\bar{X} = (L^1(x) \cap N, L^1(x^{-1}) \cap N)$ via the formula

$$K(t,f;\bar{X}) \approx K(t,f;L^1(x),L^1(x^{-1})) + \sqrt{t}\,|\int_0^{\sqrt{t}} f(s)\,ds|,$$

which shows that for $\theta \neq \frac{1}{2}$ we have that

$$(L^1(x) \cap N, L^1(x^{-1}) \cap N)_{\theta,1} = (L^1(x), L^1(x^{-1}))_{\theta,1} \cap N = L^1(x^{1-2\theta}) \cap N,$$

and for $\theta = \frac{1}{2}$ that

$$(L^1(x) \cap N, L^1(x^{-1}) \cap N)_{1/2,1} = L^1 \cap C_1 \cap N = C_1 \cap L^1,$$

where C_1 is a Cesàro function space of non-absolute type given by the norm

$$\|f\|_{C_1} = \int_0^\infty \left| \frac{1}{x} \int_0^x f(t)\,dt \right| dx.$$

In fact,

$$\begin{aligned}
\|f\|_{\bar{X}_{1/2,1}} &\approx \int_0^\infty \frac{K(t,f;\bar{X})}{\sqrt{t}}\,\frac{dt}{t} \\
&\approx \int_0^\infty \frac{K(t,f;L^1(x),L^1(x^{-1}))}{\sqrt{t}}\,\frac{dt}{t} + \int_0^\infty \left| \int_0^{\sqrt{2}} f(s)\,ds \right| \frac{dt}{t} \\
&\approx \|f\|_{L^1} + \int_0^\infty \left| \frac{1}{x} \int_0^x f(t)\,dt \right| dx = \|f\|_{L^1} + \|f\|_{C_1}.
\end{aligned}$$

In particular, interpolation of intersection property fails for $\theta = 1/2$ and $q = 1$ since

$$(L^1(x) \cap N, L^1(x^{-1}) \cap N)_{1/2,1} \neq (L^1(x), L^1(x^{-1}))_{1/2,1} \cap N = L^1 \cap N.$$

Krugljak-Maligranda-Persson [P6] also observed that the validity of the last relation is closely related to the possibility of "interpolating" certain Hardy-type integral inequalities. This paper was the starting point of several further developments of this challenging problem, e.g. by Ivanov-Kalton (2001), Kaijser-Sunehag (2002, 2003), Sunehag (2004), Astashkin (2003, 2005, 2007), Astashkin-Sunehag (2006, 2008) and Asekritova-Krugljak (2008).

Paper [P7] from 2001 is about homogenization of nonlinear monotone operators. Lions, Lukkassen[2], Persson and Wall[2] studied reiterated homogenization for nonlinear elliptic equations of the form

$$\operatorname{div} a(x, x/\varepsilon, x/\varepsilon^2, Du_\varepsilon) = f,$$

where a is periodic in the second and third variables, and monotone in the fourth one. They showed that u_ε converges weakly in $W^{1,p}(\Omega), 1 < p < \infty$, and in some multiscale sense to the solution of a limit problem. An explicit construction of the limit problem was given.

The last three authors discussed their first version of the paper with J. L. Lions and he gave them another proof using arguments known by Lions and closely equipped with the notion of two-scale convergence[3]. After that the paper was published as a joint one and just this paper has started a lot of complementary activities in Homogenization Theory.

We have also written the paper [P8], where two reiteration results of the Lions-Peetre type are proved for triples of quasi-Banach function spaces on the same measure space and which are not true, in general, for arbitrary Banach triples (Asekritova-Krugljak-Maligranda-Nikolova-Persson, 2001):

$$(\bar{X}_{\theta_0,q_0}, \bar{X}_{\theta_1,q_1}, \bar{X}_{\theta_2,q_2})_{\lambda,q} = \bar{X}_{\theta,q}, \qquad \overline{\theta} = (1-\lambda_1-\lambda_2)\overline{\theta}_0 + \lambda_1\overline{\theta}_1 + \lambda_2\overline{\theta}_2$$

and

$$((X_0, X_2)_{\alpha_0,q_0}, (X_1, X_2)_{\alpha_1,q_1})_{\mu,q} = (X_0, X_1, X_2)_{(\theta_1,\theta_2),q},$$

where

$$\theta_1 = (1-\alpha_1)\mu, \qquad \theta_2 = \alpha_0(1-\mu) + \alpha_1\mu.$$

Taking into account these theorems, it was possible to prove that interpolation of triples of Besov spaces $B^{\sigma_i}_{p_i}$, $(i = 0, 1, 2)$, produces Lorentz-Besov spaces $B^{\sigma,q}(L_{p,q})$, which for couples of Besov spaces is rather complicated, except for the diagonal case. Another application was given in connection to a generalization of the Stein-Weiss interpolation theorem known for Lebesgue spaces $L_{p_i}(w_i\, d\mu)$, $(i = 0, 1)$, with change of measures to the corresponding Lorentz spaces $L_{p_i,q_i}(w_i\, d\mu)$, $(i = 0, 1)$. The result proved here shows that these spaces are not weighted Lorentz spaces but block-Lorentz spaces (weighted Herz-Lorentz spaces). In particular, the results obtained here show that for some problems in analysis the three-space real interpolation approach is really more useful than the usual real interpolation between couples.

The well-known theorem states that the weighted Hardy inequality

$$\left(\int_0^\infty \left(\int_0^x f(t)\, dt\right)^q u(x)\, dx\right)^{1/q} \le C\left(\int_0^\infty f(x)^p v(x)\, dx\right)^{1/p}$$

[2]Professors Dag Lukkassen and Peter Wall have both obtained their PhD in mathematics under the supervision of LEP.

[3]It seems as J. L. Lions early knew about arguments which later on led to the formal definition of two-scale convergence introduced in 1989 by G. Nguetseng.

holds for any $f \geq 0$ with positive weights u, v if and only if the Muckenhoupt function

$$A_M(x) := \left(\int_x^\infty u(t)\, dt\right)^{1/q} \left(\int_0^x v(t)^{1-p'}\, dt\right)^{1/\max(p',q')}$$

is bounded in the case $1 \leq p \leq q < \infty$ and belongs to $L^r((0,\infty), v(x)^{1-p'}\, dx)$ in the case $1 \leq q < p < \infty$, where $\frac{1}{r} = \frac{1}{q} - \frac{1}{p}$ (cf. Kufner-Persson [4] or Kufner-Maligranda-Persson [P7]).

In the paper [P9] Persson-Stepanov were trying to prove the weighted Carleman inequality by limiting process in the corresponding scale of Hardy type inequalities and then they discovered that in the above classical weighted Hardy inequality it is possible to take instead of the "Muckenhoupt function" $A_M(x)$ another one – the "Persson-Stepanov function" – defined by

$$A_{PS}(x) = \left(\int_0^x v(t)^{1-p'} dt\right)^{-1/\max(p,q)} \left(\int_0^x u(t) (\int_0^t v(s)^{1-p'}\, ds)^q\, dt\right)^{1/q}$$

and the same result will be true in both the cases $1 \leq p \leq q < \infty$ and $1 \leq q < p < \infty$.

LEP has later on together with coauthors proved that also these conditions are not unique and can in fact be replaced by infinite many conditions, namely four different scales of conditions.

It was a very special time when we were writing the historical paper [P10]. LEP wrote a first version of this paper. He decided in each Section to only use the information, which we judged could be available for Hardy that year. LEP has pronounced to me that sometimes during this summer when he awaked he not only felt like Hardy, he even at the first moment hesitated if he in fact was Hardy!

In this paper (which I know LEP likes very much) we described the dramatic prehistory of the classical Hardy inequality in detail. The most important contributors here, except Hardy himself, were E. Landau, G. Pólya, I. Schur, and M. Riesz. Hardy finally proved his famous inequality in 1925, but the research activities (motivated by finding a simple proof of the Hilbert inequality) started already around 1915 and the formulations appeared already in 1920 and 1921 in the correspondence with Schur and Landau. We can see from this investigation that the Hardy inequality could have been given the name Riesz or Landau-Riesz or Hardy-Landau-Riesz inequality. The reviewer of this paper, Peter S. Bullen, wrote the following in MR2256532 (2007e:26002):

> *This excellent paper should be read by all interested in real analysis, in particular those working in inequalities. The development of the Hardy inequalities over the decade 1915–1925 is given in detail.*

Finally, I would like to shortly mention about our two jointly written books [6] and [7]. The first book [6] was published in 2006 by Larsson-Maligranda-Pečarić-Persson and here is collected on 215 pages the historical development of the Carlson inequalities with many generalizations and variations, e.g. those in the PhD thesis by L. Larsson[4]. It contains almost all known results concerning these inequalities, all known techniques of proofs and a number of open questions suitable for further research. This book is of interest for mathematicians working in several disciplines within analysis and its applications (e.g. theory of interpolation, harmonic analysis, function spaces, inequalities, etc.)

The second book [7] appeared in 2007 and the authors Kufner-Maligranda-Persson collected on 162 pages in the first half part of the book some of the most important points of the rich history of development of Hardy type inequalities and in the other half numerous topics connected with the Hardy inequality with proofs, and including almost all references. The reviewer Benjamin Muckenhoupt wrote the following in MR2351524 (2008j:26001):

> *Overall the book contains a good introduction to the history and many current results concerning Hardy's inequality. It also contains some explicit unsolved problems and many implicit ones. It should be of interest to anyone who has worked with this famous inequality.*

Let me finally present a nonstandard proof that LEP really is a mathematician. LEP visited sometimes professor Fernando Cobos in Madrid. Once after his stay in a hotel in Madrid, Fernando noted that he forgot all the clothes he had in the wardobe. Fernando phoned to Lars-Erik and asked:

frankly speaking Lars-Erik, did you arrive completely nude at home?
LEP answered that he had still not noticed that, but he was a little suspicious that his *"luggage on the way back was so light"* and he could not understand why.

8. Publications

LEP has written 16 books (all together with other authors), namely 8 monographs, 5 conference proceedings, where he was one of the editors, and 3 textbooks. Moreover, he has written over 200 papers (several of them are written jointly with other authors). I present here only his 10 most cited papers (in order of publication year).

MONOGRAPHS, BOOKS

1. *The Homogenization Method. An Introduction*, Studentlitteratur, Lund 1993, 86 pages (together with Leif Persson, Nils Svanstedt and John Wyller).
2. *Integral Inequalities with Weights*, Academy of Sciences of the Czech Republic 2000, 211 pages (together with Alois Kufner).
3. *Convex Functions – Basic Theory and Applications*, Universitaria Press 2003, 185 pages (together with Constantin P. Niculescu).
4. *Weighted Inequalities of Hardy Type*, World Scientific Publishing Co., Inc., River Edge, NJ 2003, 375 pages (together with Alois Kufner).
5. *Convex Functions and their Applications. A Contemporary Approach*, Canad. Math. Series Books in Mathematics, Springer, New York 2006, 271 pages (together with Constantin P. Niculescu).

[4]LEP was supervisor for Leo Larsson.

6. *Multiplicative Inequalities of Carlson Type and Interpolation*, World Scientific, Singapore 2006, 215 pages (together with Leo Larsson, Lech Maligranda and Josip E. Pečarić).
7. *The Hardy Inequality – About its History and Some Related Results*, Vydavatelski Servis Publishing House, Pilzen 2007, 162 pages (together with Alois Kufner and Lech Maligranda).
8. *Weighted Norm Inequalities for Integral Transforms with Product Kernels*, Nova Scientific Publishers, Inc., New York 2009, 329 pages (together with Alexander Meshki and Vakhtang Kokalishvili).

CONFERENCE PROCEEDINGS

9. *Analysis, Algebra, and Computers in Mathematical Research*, Proceedings of the Twenty-first Nordic Congress of Mathematicians held at Luleå University of Technology, Luleå, 1992, Lecture Notes in Pure and Applied Mathematics 156, Marcel Dekker, New York 1994, 418 pages (Editor together with Mats Gyllenberg).
10. *Selected Topics in Mathematics*, Proceedings of the first Nordic Summer School for Female PhD Students of Mathematics in Luleå (June 1996), Luleå University 1997, 110 pages (Editor together with Anna Klisińska).
11. *Difference and Differential Inequalities*, Proceedings of the International Workshop in Istanbul, Istanbul, Turkey, July 3-7 1996, in: Mathematical Inequalities and Applications 1(1998), No. 3, 347-461 (Editor together with Rawi P. Agarwal and A. Zafer).
12. *Function Spaces and Applications*, Proceedings from the International Conference on Function Spaces and Applications to Partial Differential Equations held at the University of Delhi, Delhi, December 15–19, 1997, Narosa Publishing House, New Delhi 2000, 286 pages (Editor together with David E. Edmunds, Pawan K. Jain, Pankaj Jain, Alois Kufner and Saburou Saitoh).
13. *Function Spaces, Interpolation Theory and Related Topics*, Proceedings of the International Conference in honour of Jaak Peetre on his 65th birthday, held at Lund University, Lund, August 17–22, 2000, Walter de Gruyter, Berlin 2002, 474 pages (Editor together with Michael Cwikel, Miroslav Englis, Alois Kufner and Gunnar Sparr).

TEXTBOOKS

14. *Calculus of Several Variables*, Studentlitteratur Publ., Lund 1990, 472 pages (together with Andrejs Dunkels, Håkan Ekblom, Torbjörn Hedberg and Reinhold Näslund).
15. *Linear Algebra*, Studentlitteratur Publ., Lund 1990, 502 pages (together with Lennart Andersson, Anders Grennberg, Torbjörn Hedberg, Reinhold Näslund and Björn von Sydow).
16. *Linear Algebra with Geometry*, Studentlitteratur Publ., Lund 1999, 487 pages (together with Lennart Andersson, Anders Grennberg, Torbjörn Hedberg, Renhold Näslund and Inge Söderqvist).

TEN SELECTED PAPERS

[P1] *Interpolation with a parameter function*, Math. Scand. **59** (1986), no. 2, 199–222.

[P2] *Generalized duality of some Banach function spaces*, Indag. Math. **51** (1989), 323–338 (joint with L. Maligranda).

[P3] *A Carlson type inequality with blocks and interpolation*, Studia Math. **104** (1993), no. 2, 161–180 (joint with N. Ya. Kruglyak and L. Maligranda).

[P4] *Distribution and rearrangement estimates of the maximal function and interpolation*, Studia Math. **124** (1997), 107–132 (joint with I. U. Asekritova, N. Ya. Krugljak and L. Maligranda).

[P5] *Convexity, concavity, type and cotype of Lorentz spaces*, Indag. Math. (N.S.) **9** (1998), no. 3, 367–382 (joint with A. Kamińska and L. Maligranda).

[P6] *The failure of the Hardy inequality and interpolation of intersections*, Arkiv Mat. **37** (1999), 323–344 (joint with N. Krugljak and L. Maligranda).

[P7] *Reiterated homogenization of nonlinear monotone operators*, Chinese Ann. Math. Ser. B **22** (2001), no. 1, 1–12 (joint with J. L. Lions, D. Lukkassen and P. Wall).

[P8] *Lions-Peetre reiteration formulas for triples and their applications*, Studia Math. **145** (2001), 219–254 (joint with I. Asekritova, N. Krugljak, L. Maligranda and L. Nikolova).

[P9] *Weighted integral inequalities with the geometric mean operator*, J. Inequal. Appl. **7** (2002), no. 5, 727–746 (joint with V. D. Stepanov).

[P10] *The pre-history of the Hardy inequality*, Amer. Math. Monthly **113** (2006), 715–732 (joint with A. Kufner and L. Maligranda).

FINAL REMARK

For more information about the publications of LEP and other information about him and his work I refer to his home page at:

`http://www.ltu.se/mat/staff/larserik`

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