Banach J. Math. Anal. 1 (2007), no. 2, 252–260

BANACH JOURNAL OF MATHEMATICAL ANALYSIS
ISSN: 1735-8787 (electronic)
http://www.math-analysis.org

AN INTERVIEW WITH THEMISTOCLES M. RASSIAS

PER ENFLO[1] AND MOHAMMAD SAL MOSLEHIAN[2]*

ABSTRACT. While the authors were visiting Athens in 2006 and 2005, respectively, they interviewed Professor Themistocles M. Rassias concerning his contributions to Mathematics. This article presents those interviews.

1. INTRODUCTION

Professor Themistocles M. Rassias has made fundamental contributions in mathematical analysis, geometry, topology and their applications. His works are known in the field of mathematical analysis with the terms "Hyers–Ulam–Rassias stability" (first appeared in [11]) or "Cauchy–Rassias stability" (first appeared in [23, 32]) and in geometry with the term "Aleksandrov–Rassias problem" (first appeared in [63]). He has lectured extensively and conducted research in various academic institutions in Europe and North America. He published more than 200 papers, wrote six research books and edited 24 volumes on different subjects in mathematics with many citations to them (see [1, 7, 15, 20, 26, 48]) and also his selected publications. He has served as a member of the Editorial Board of more than 40 journals published in Europe, USA and Asia, which gives him a global view of the current research. He is Professor at the National Technical University of Athens, Greece.

While the authors were visiting Athens in 2006 and 2005, respectively, they interviewed Professor Themistocles M. Rassias concerning his contributions to Mathematics. This article presents those interviews.

Date: Received: July 10, 2006; Accepted: 25 September 2007.

* Corresponding author.

2000 *Mathematics Subject Classification.* Primary 01A99; Secondary 01A60, 01A61.

Key words and phrases. Hyers–Ulam–Rassias stability, Aleksandrov–Rassias problem, mathematical analysis.

2. Interview

• Please tell us about yourself and your early education. How did you get interested in mathematics? Where, when and how you got your Ph.D.?

⋆ I was born in the year 1951 in the village Pellana about 25 kilometers far from Sparta in the southern part of Greece. I finished the Elementary School in Pellana where it happened to have two excellent teachers. I remember myself having a passion for properties of numbers and for geometry. I continued in the High School of Kastorion, a village almost six kilometers far my home. I finished the final year in the High School in Sparta. In both these schools, I had the priviledge to have excellent teachers in most subjects but especially in mathematics. After I finished High School, I entered the Mathematics Department of the University of Thessaloniki where I studied for the first two years of the four years degree program.
At the age of 19, I published "Modern Mathematics". In that year I started studying by myself some more advanced mathematics. When I finished my second year of studies in Thessaloniki I continued my studies in USA. I started my studies for a few months at the George Washington University in Washington DC. There the Mathematics Department decided to register me straight to the Graduate Program in Mathematics before I finish the two remaining years of study for the Bachelor degree. Next, I continued my graduate studies at the University of California at Berkeley,where I received my Ph.D. in mathematics in June 1976 under the direction of Stephen Smale (Fields Medalist, Moscow 1966). My academic advisor at Berkeley was S.S. Chern.

• Can you tell us something about the professors who you participated in their classes and Seminars? What were your impressions of them?

⋆ At Berkeley I attended graduate courses and Seminars from the following Professors: S.S. Chern, M. Freedman, G. Hile, H. Helson, M. Hirsch, T. Kato, J. Kelley, M. Loeve, C. Moore, M. Rieffel, S. Smale and others. They inspired me for my work of a lifetime.

If I could mention some of my mathematical heroes from my graduate and postodoctoral studies at Berkeley I would definitely mention S. Smale, S.S. Chern, C.B. Morrey,Jr., T. Kato and M. Freedman (Fields Medalist, Berkeley 1986). In the rest of my academic life I communicated with and learned from L. Ahlfors (Fields Medalist, Oslo 1936), G.G. Birkhoff, R. Bott, H. Cartan, J. Dieudonne, S. Donaldson (Fields Medalist, Berkeley 1986), D.H. Hyers, J. Leray, J. Lions, M. Morse, D.J. Struik, S.M. Ulam, Van Der Waerden and O. Zariski.

• What are your honors and awards?

⋆ I feel embarassed talking about myself.

• Please do mention without inhibition some of your honors to the extent they reflect on your work and your recognition.

⋆ "Membership" at the School of Mathematics of the Institute for Advanced Study at Princeton for the academic years 1977–1978 and 1978–1979 which I did not accept for family reasons; "Research Associate" at the Department of Mathematics of Harvard University (April 1980) invited by Professor Raoul Bott; "Visiting Research Professor" at the Department of Mathematics of the Massachusetts Institute of Technology (April 1980) invited by Professor F.P. Peterson; "Accademico Ordinario" of the Accademia Tiberina Roma (since November, 1987); "Fellow" of the Royal Astronomical Society of London (since 1991); "Teacher of the Year" (1985–1986, 1986–1987); "Outstanding Faculty Member" (1989–1990, 1990–1991, 1991–1992).
However, I feel that the lasting award granted to me for my research is the standardized now "terminology" in mathematical analysis "Hyers–Ulam–Rassias stability" or "Cauchy–Rassias stability" and in geometry the term "Aleksandrov–Rassias problem".

Themistocles M. Rassias

• What are your most important achievement in mathematics? What are your favorite papers? How many coauthors have you had?

⋆ It is hard to say what are my most important achievements in mathematics since as you know mathematical results and their proofs or new concepts are recognized after long time (or never recognized). Mathematics is interrelated. I guess my articles leading to the concepts of Hyers–Ulam–Rassias stability (Cauchy–Rassias stability) and Aleksandrov–Rassias problem have received the most acclaim.

As for coauthors, I feel fortunate that I had the opportunity to collaborate with about 50 excellent mathematicians from several countries.

• Your name is equipped in functional equations with the terms "Hyers–Ulam–Rassias stability" or "Cauchy–Rassias stability" and in geometry of metric spaces with the term "Aleksandrov–Rassias problem". Would you please briefly explain them and give us a brief history of them?

$\star$ I should quote S.M. Ulam [62, Page 63], for very general functional equations, one can ask the following question: "When is it true that the solution of an equation (e.g. f(x+y)=f(x)+f(y)) differing slightly from a given one, must of necessity be close to the solution of the given equation?" Similarly, if we replace a given functional equation by a functional inequality, when can one assert that the solutions of the inequality lie near the solutions of the strict equation? The stability problem of functional equations indeed originated from the question of Ulam concerning the stability of group homomorphisms. In the year 1941 a partial solution of the problem was provided by D.H. Hyers [13] in the case of an approximate homomorphism between Banach spaces in the context of the Cauchy difference. Hyers' theorem was generalized for approximate additive mappings by T. Aoki [2] and for approximate linear mappings by me [38] by considering the unbounded Cauchy difference inequality

$$\|f(x+y)-f(x)-f(y)\| \leq \varepsilon(\|x\|^p+\|y\|^p)\,, \tag{2.1}$$

where $\varepsilon > 0$ and $p \in [0,1)$ are fixed numbers. In 1990, during the 27th International Symposium on Functional Equations I asked the question whether such a theorem can also be proved for $p \geq 1$; cf. [39]. In 1991, Z. Gajda [10] following the same approach as in [38] gave an affirmative solution to this question for $p > 1$. It was proved by Z. Gajda [10], as well as by P. Šemrl and myself [55] that one cannot prove a similar theorem when $p = 1$.

In fact, with my paper [38], I introduced the concept of the unbounded Cauchy difference and thus the Hyers–Ulam stability problem for the linear mappings between Banach spaces was brought in a new general context. This has led to the stability phenomenon that is called the Hyers–Ulam–Rassias stability or Cauchy–Rassias stability. Subsequently, a number of mathematicians following the spirit of my approach [38] for the unbounded Cauchy difference obtained several similar results. For example, in 1982, J.M. Rassias [37] proved a similar theorem in which he replaced the factor $\|x\|^p+\|y\|^p$ by $\|x\|^p\cdot\|y\|^q$ for $p, q \in \mathbb{R}$ with $p+q \neq 1$. In 1994, P. Găvruţa [11] provided a further generalization of my theorem in which he replaced the bound $\varepsilon(\|x\|^p+\|y\|^p)$ in (2.1) by a general control function $\varphi(x,y)$. In 1996, G. Isac and myself [18] were the first to apply the Hyers–Ulam–Rassias stability theory to prove new fixed point theorems and obtained some applications in nonlinear analysis.

During the last three decades several stability problems of functional equations have been investigated by a number of mathematicians worldwide; see [7, 15, 20, 41, 48] and the references therein. I feel grateful to both D.H. Hyers and

S.M. Ulam for their encouragement they have provided me especially at the very beginning when I proved my theorem in 1977 (that was published later in 1978).

The Aleksandrov–Rassias problem asks to find conditions under which a transformation from a metric space (X, d) to a metric space (Y, ρ) which preserve one distance (i.e. for some constant c, $d(x, y) = c$ implies $\rho(f(x), f(y)) = c$ for all $x, y \in X$), or possibly two distances is an isometry. This problem can be formulated for transformations between Euclidean or non-Euclidean spaces or other more general spaces. It leads to other problems of nonexpanding or non-shrinking distances of transformations which are studied in geometric optics as well as in quantum field theory. (See [61] for more information.)

• Can you tell us you prefer to teach or to do research? What are your goals in teaching undergraduates and in supervising graduate students?

⋆ I prefer to do both research and teaching. When I prove a new theorem or introduce a new concept this enlightens my brain but when I teach mathematics, I feel happy. There are no border lines in research mathematics and in teaching. It is always nice to try to unify mathematical knowledge and thinking as well as creativity. When I teach undergraduate students, I always start with examples and counterexamples. After the students start to understand and appreciate the main ideas, I define abstract concepts but there things go smoother and the road to mathematical reality seems comprehensible and more exciting. The emphasis is on mathematical thinking and creativity and not on memorization. This is the key point!.

As for graduate students the main thing is to teach them how to think, how to search, how to use the library and to feel more self-reliant. It is essential to give them encouragement to go further in the vast space of the unknown and to fight for original thinking. Of course when they prove their first proposition or theorem or when they give a good example it is essential to also help them how to write it down understandably.

• Many people think of mathematics as a hard subject. Is mathematics really hard? Kindly explain.

⋆ Mathematics as a science from the most elementary concepts up to the more complex problems and theories is often treated badly by teachers and this is often because of lack of mathematical maturity of the teacher. It is essential to approach the subject through concrete examples (and with a large variety) so that the theoretical concepts will come later as a natural consequence. Mathematics is NOT a hard subject, BUT “educators” make it hard.

• Any comments on the relation between pure and applied mathematics? Why mathematics can be applied to other areas of sciences or Technology?

⋆ There has been a lot of prejudice regarding the relationship of pure and applied mathematics. Since World War II very applied mathematical problems have necessitated the most advanced pure mathematical constructs. This has proven to be also the case in the relationships between Theoretical Physics, Theoretical Biology and applications. Therefore the distinction of pure and applied is more an organizational.

• I know you have two children. Were your children interested in mathematics?

⋆ Both my children had an inclination in mathematics and a lot of interest in it. My son Michael as a High School student during the period 14-16 years of age he was awarded twice the Gold Medal of the National Mathematical Olympiad in Greece in the years 2002 and 2003, respectively. He was also awarded the Silver Medal of the Balkan Mathematical Olympiad in 2002, the Silver Medal of the International Mathematical Olympiad in Tokyo, Japan, in 2003, as well as three First Prizes from the Jozelf Wildt International Mathematical Competitions in the years 2004, 2005 and 2006, respectively.
My daughter Matina is currently working for her Ph.D. in Architecture at Cambridge University. Her facility in quantitative methods is apparent in her work.

• Mathematics in ancient Greece was very impressive, as we know from History. What is your feeling?

⋆ Geometry and number theory have been monumental and have influenced new mathematics all along. Indeed, mathematics was in the cross section of all different "roads" of knowledge from every subject.

References

[1] M. Amyari and M.S. Moslehian, *Approxiamtely ternary semigroup homomorphisms*, Lett. Math. Phys. **77** (2006), 1–9.

[2] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan **2** (1950) 64–66.

[3] C.Baak, D.-H. Boo and Th.M. Rassias, *Generalized additive mapping in Banach modules and isomorphisms between C^*-algebras*, J. Math. Anal. Appl. **314** (2006), 150–161.

[4] B. Belaid, E. Elhoucien and Th.M. Rassias, *On the generalized Hyers–Ulam stability of the quadratic functional equation with a general involution*, Nonlinear Funct. Anal. Appl. (to appear).

[5] B. Belaid, E. Elhoucien and Th.M. Rassias, *On the Hyers–Ulam stability of approximately Pexider mappings*, Math. Ineq. Appl. (to appear).

[6] M. Craioveanu, M. Puta and Th.M. Rassias, *Old and New Aspects in Spectral Geometry*, Kluwer Academic Publishers, Dordrecht, Boston, London, 2001.

[7] S. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific, New Jersey, London, Singapore, Hong Kong, 2002.

[8] S.S. Dragomir and Th.M. Rassias, *A mapping associated with Jensen's inequality and applications*, Bull. Math. Soc. Sci. Math. Roumanie (N.S.) **44** (92) (2001), no. 2, 155–164.

[9] V. Faĭziev, Th.M. Rassias and P.K. Sahoo, *The space of* (ψ,γ)*-additive mappings on semigroups* Trans. Amer. Math. Soc. **354** (2002), no. 11, 4455–4472.

[10] Z. Gajda, *On stability of additive mappings*, Internat. J. Math. Math. Sci. **14** (1991), 431–434.

[11] P. Găvruţa, *A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.

[12] H. Haruki and Th.M. Rassias, *A new characterization of Möbius transformations by use of Apollonius hexagons*, Proc. Amer. Math. Soc. **128** (2000), 2105–2109.

[13] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A. **27** (1941), 222–224.

[14] D.H. Hyers, G. Isac and Th.M. Rassias, *Topics in Nonlinear Analysis and Applications*, World Scientific Publishing Co., Singapore, New Jersey, London, 1997.

[15] D.H. Hyers, G. Isac and Th.M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, 1998.

[16] D.H. Hyers, G. Isac and Th.M. Rassias, *On the asymptoticity aspect of Hyers-Ulam stability of mappings*, Proc. Amer. Math. Soc. **126** (1998), 425–430.

[17] D.H. Hyers and Th.M. Rassias, *Approximate homomorphisms*, Aequationes Math. **44** (1992), 125–153.

[18] G. Isac and Th.M. Rassias, *Stability of* ψ*-additive mappings: Applications to nonlinear analysis*, Internat. J. Math. Math. Sci. **19** (1996), 219–228.

[19] G. Isac and Th.M. Rassias, *On the Hyers-Ulam stability of* ψ*-additive mappings*, J. Approx. Theory **72** (1993), 131–137.

[20] S.-M. Jung, *Hyers–Ulam–Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Palm Harbor, 2001.

[21] S.-M. Jung and Th.M. Rassias, *Ulam's problem for approximate homomorphisms in connection with Bernoulli's differential equation*, Appl. Math. Comput. **187** (2007), 223–227.

[22] S.-M. Jung and Th.M. Rassias, *Generalized Hyers–Ulam stability of Riccati differential equation*, Math. Inequal. Appl. (to appear).

[23] J.-R. Lee and D.Y. Shin, *On the Cauchy–Rassias stability of the Trif functional equation in* C^**-algebras*, J. Math. Anal. Appl. **296** (2004), 351–363.

[24] B. Mielnik and Th.M. Rassias, *On the Aleksandrov problem of conservative distances*, Proc. Amer. Math. Soc. **116** (1992), 1115–1118.

[25] G.V. Milovanović, D.S. Mitrinović and Th.M. Rassias, *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, World Scientific Publishing Co., Inc., River Edge, NJ, 1994.

[26] M.S. Moslehian, *Ternary derivations, stability and physical aspects*, Acta Appl. Math. (to appear).

[27] M.S. Moslehian and Th.M. Rassias, *Orthogonal stability of additive type equations*, Aequationes Math., **73** (2007) 249–259.

[28] M. S. Moslehian and Th. M. Rassias, *Stability of functional equations in non-Arhimedian spaces*, Appl. Anal. Disc. Math. **1** (2007), 325–334.

[29] M.S. Moslehian and Th.M. Rassias, *Generalized Hyers–Ulam stability of mappings on normed Lie triple systems*, Math. Inequal. Appl. (to appear).

[30] M.A. Noor, K.I. Noor and Th.M. Rassias, *Some aspects of variational inequalities* J. Comput. Appl. Math. **47** (1993), 285–312.

[31] M.A. Noor, K.I. Noor and Th.M. Rassias, *Set-valued resolvent equations and mixed variational inequalities* J. Math. Anal. Appl. **220** (1998), 741–759.

[32] C.-G. Park, *Generalized quadratic mappings in several variables*, Nonlinear Anal. **57** (2004), 713–722.
[33] C.-G. Park and Th.M. Rassias, *Hyers–Ulam stability of a generalized Apollonius type quadratic mapping*, J. Math. Anal. Appl. **322** (2006), 371–381.
[34] C.-G. Park and Th.M. Rassias, *The N-isometric isomorphisms in linear n-normed C^*-algebras*, Acta Math. Sinica (English Series), **22** (2006), 1863–1890.
[35] C.-G. Park and Th.M. Rassias, *Inequalities in additive N-isometries on linear N-normed Banach spaces*, J. Inequal. Appl., **2007** (2007), Article ID 70597, pp.1–12.
[36] A. Prastaro and Th.M. Rassias, *Ulam stability in geometry of PDE's*, Nonlinear Funct. Anal. Appl. **8** (2003), 259–278.
[37] J. M. Rassias, *On approximation of approximately linear mappings by linear mappings*, J. Funct. Anal. **46** (1982), 126–130.
[38] Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. **72** (1978), 297–300.
[39] Th.M. Rassias, *Problem 16 ; 2, Report of the 27th International Symp.on Functional Equations*, Aequationes Math. **39** (1990), 292–293; 309.
[40] Th.M. Rassias, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl. **251** (2000), 264–284.
[41] Th.M. Rassias (ed.), *Functional Equations, Inequalities and Applications*, Kluwer Academic Publishers, Dordrecht, Boston and London, 2003.
[42] Th.M. Rassias, *Is a distance one preserving mapping between metric spaces always an isometry?* Amer. Math. Monthly **90** (1983), 200.
[43] Th.M. Rassias, *On the stability of mappings*, Rendiconti del Seminario Matematico e Fisico di Milano **58** (1988), 91–99.
[44] Th.M. Rassias, *On a modified Hyers–Ulam sequence*, J. Math. Anal. Appl. **158** (1991), 106–113.
[45] Th.M. Rassias, *On the stability of the quadratic functional equation and its applications*, Studia Univ.Babes-Bolyai Math. **43** (1998), 89–124.
[46] Th.M. Rassias, *On the stability of functional equations originated by a problem of Ulam*, Mathematica. **44**(67)(1)(2002), 39–75.
[47] Th.M. Rassias, *The problem of S.M.Ulam for approximately multiplicative mappings*, J. Math. Anal. Appl. **246**(2) (2000), 352–378.
[48] Th.M. Rassias, *On the stability of functional equations and a problem of Ulam*, Acta Appl. Math. **62** (2000), 23–130.
[49] Th.M. Rassias, *Stability of the generalized orthogonality functional equation*, in: Inner Product Spaces and Applications, Addison Wesley Longman, Pitman Research Notes in Mathematics Series No. 376 (ed. Th.M. Rassias), 1997, pp. 219–240.
[50] Th.M. Rassias, *Stability and set-valued functions*, in: Analysis and Topology (ed.Th.M. Rassias), World Scientific Publishing Co., 1998, pp. 585–614.
[51] Th.M. Rassias, *Properties of isometric mappings*, J. Math. Anal. Appl. **235** (1999),108–121.
[52] Th.M. Rassias, *Isometries and approximate isometries*, Internat J. Math. Math. Sci. **25**(2) (2001), 73–91.
[53] Th.M. Rassias, *On the stability of minimum points*, Mathematica 45(68)(1)(2003),93-104.
[54] Th.M. Rassias, *On the A.D. Aleksandrov problem of conservative distances and the Mazur–Ulam theorem*, Nonlinear Anal., **47**(4) (2001), 2597–2608.
[55] Th.M. Rassias and P. Šemrl, *On the behaviour of mappings which do not satisfy Hyers–Ulam stability*, Proc. Amer. Math. Soc. **114** (1992), 989–993.
[56] Th.M. Rassias and P. Šemrl, *On the Mazur–Ulam theorem and the Aleksandrov problem for unit distance preserving mappings*, Proc. Amer. Math. Soc. **118** (1993), 919–925.
[57] Th.M. Rassias and P. Šemrl, *On the Hyers–Ulam stability of linear mappings*, J. Math. Anal. Appl. **173** (1993), 325–338.

[58] Th.M. Rassias and J. Simsa, *Finite Sums Decompositions in Mathematical Analysis*, John Wiley & Sons, Wiley–Interscience Series in Pure and Applied Mathematics, 1995.
[59] Th.M. Rassias and J. Tabor, *What is left of Hyers–Ulam stability?*, J. Natur. Geom. **1** (1992), 65–69.
[60] Th.M. Rassias and S. Xiang, *On Mazur–Ulam theorem and mappings which preserve distances*, Nonlinear Funct. Anal. Appl. **5** (2000), 61–66.
[61] L. Tan and S. Xiang, *On the Aleksandrov–Rassias problem and the Hyers–Ulam–Rassias stability problem*, Banach J. Math. Anal. **1** (2007), 11–22.
[62] S.M. Ulam, *Problems in Modern Mathematics*, Chapter VI, Science Editions, Wiley, New York, 1964.
[63] S. Xiang, *On the Aleksandrov–Rassias problem for isometric mappings* Functional equations, inequalities and applications, 191–221, Kluwer Acad. Publ., Dordrecht, 2003.

[1] Department of Mathematics, Kent State University, Kent, OH 44242 USA.
E-mail address: enflo@math.kent.edu

[2] Department of Mathematics, Ferdowsi University, P.O. Box 1159, Mashhad 91775, Iran.
E-mail address: moslehian@ferdowsi.um.ac.ir and moslehian@ams.org