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## ON LEBESGUE TYPE DECOMPOSITION FOR COVARIANT COMPLETELY POSITIVE MAPS ON $C^*$ -ALGEBRAS

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**ABSTRACT.** We show that there is an affine order isomorphism between completely positive maps from a  $C^*$ -algebra  $A$  to the  $C^*$ -algebra  $L(H)$  of all bounded linear operators on a Hilbert space  $H$ ,  $u$ -covariant with respect to a  $C^*$ -dynamical system  $(G, \alpha, A)$  and  $u$ -covariant completely positive maps from the crossed product  $A \times_{\alpha} G$  to  $L(H)$ , which preserves the Lebesgue decomposition.

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