ON A REVERSE OF ANDO–HIAI INEQUALITY

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This paper is dedicated to Professor Lars-Erik Persson

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ABSTRACT. In this paper, we show a complement of Ando–Hiai inequality: Let A and B be positive invertible operators on a Hilbert space H and $\alpha \in [0,1]$. If $A \, \sharp_\alpha \, B \leq I$, then

$$A^r \, \sharp_\alpha \, B^r \leq \|(A \, \sharp_\alpha \, B)^{-1}\|^{1-r} I \qquad \text{for all } 0 < r \leq 1,$$

where I is the identity operator and the symbol $\| \cdot \|$ stands for the operator norm.

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