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MATRIX ORDER IN BOHR INEQUALITY FOR OPERATORS

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This paper is dedicated to Professor Lars-Erik Persson

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Abstract. The classical Bohr inequality says that $|a+b|^2 \le p|a|^2+q|b|^2$ for all scalars a, b and p, $q > 0$ with $\frac{1}{p}+\frac{1}{q}=1$. The equality holds if and only if $(p-1)a = b$. Several authors discussed operator version of Bohr inequality. In this paper, we give a unified proof to operator generalizations of Bohr inequality. One viewpoint of ours is a matrix inequality, and the other is a generalized parallelogram law for absolute value of operators, i.e., for operators A and B on a Hilbert space and $t \neq 0$,

$$|A-B|^2 + \frac{1}{t}|tA+B|^2 = (1+t)|A|^2 + (1+\frac{1}{t})|B|^2.$$

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