

Banach J. Math. Anal. 3 (2009), no. 2, 16–27

BANACH JOURNAL OF MATHEMATICAL ANALYSIS ISSN: 1735-8787 (electronic) http://www.math-analysis.org

## $E_0\mbox{-}{\rm SEMIGROUPS}$ FOR CONTINUOUS PRODUCT SYSTEMS: THE NONUNITAL CASE

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Communicated by M. Frank

ABSTRACT. Let  $\mathcal{B}$  be a  $\sigma$ -unital  $C^*$ -algebra. We show that every strongly continuous  $E_0$ -semigroup on the algebra of adjointable operators on a full Hilbert  $\mathcal{B}$ -module E gives rise to a full continuous product system of correspondences over  $\mathcal{B}$ . We show that every full continuous product system of correspondences over  $\mathcal{B}$  arises in that way. If the product system is countably generated, then E can be chosen countably generated, and if E is countably generated, then so is the product system. We show that under these countability hypotheses there is a one-to-one correspondence between  $E_0$ -semigroups up to stable cocycle conjugacy and continuous product systems up to isomorphism. This generalizes the results for unital  $\mathcal{B}$  to the  $\sigma$ -unital case.

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Date: Received: 13 January 2009; Accepted: 7 April 2009.

<sup>2000</sup> Mathematics Subject Classification. Primary 46L55; Secondary 46L53, 46L08.

Key words and phrases. Quantum probability, quantum dynamic, product system, Hilbert module, classification.