



## A GENERALIZATION OF THE WEAK AMENABILITY OF BANACH ALGEBRAS

A. BODAGHI<sup>1</sup>, M. ESHAGHI GORDJI<sup>2\*</sup> AND A. R. MEDGHALCHI<sup>3</sup>

Communicated by N. Grønbæk

ABSTRACT. Let  $A$  be a Banach algebra and let  $\varphi$  and  $\psi$  be continuous homomorphisms on  $A$ . We consider the following module actions on  $A$ ,

$$a \cdot x = \varphi(a)x, \quad x \cdot a = x\psi(a) \quad (a, x \in A).$$

We denote by  $A_{(\varphi, \psi)}$  the above  $A$ -module. We call the Banach algebra  $A$ ,  $(\varphi, \psi)$ -weakly amenable if every derivation from  $A$  into  $(A_{(\varphi, \psi)})^*$  is inner. In this paper among many other things we investigate the relations between weak amenability and  $(\varphi, \psi)$ -weak amenability of  $A$ . Some conditions can be imposed on  $A$  such that the  $(\varphi'', \psi'')$ -weak amenability of  $A^{**}$  implies the  $(\varphi, \psi)$ -weak amenability of  $A$ .

<sup>1</sup>SCIENCE AND RESEARCH BRANCH, ISLAMIC AZAD UNIVERSITY (IAU), TEHRAN, IRAN.

*E-mail address:* [abasalt\\_bodaghi@yahoo.com](mailto:abasalt_bodaghi@yahoo.com)

<sup>2</sup> DEPARTMENT OF MATHEMATICS, SEMNAN UNIVERSITY, P. O. BOX 35195-363, SEMNAN, IRAN.

*E-mail address:* [madjid.eshaghi@gmail.com](mailto:madjid.eshaghi@gmail.com), [maj\\_ess@yahoo.com](mailto:maj_ess@yahoo.com)

<sup>3</sup> DEPARTMENT OF MATHEMATICS, TARBIAT MOALLEM UNIVERSITY, TEHRAN, IRAN.

*E-mail address:* [a.medghalchi@saba.tmu.ac.ir](mailto:a.medghalchi@saba.tmu.ac.ir)

---

*Date:* Received: 31 July 2008; Revised: 17 November 2008; Accepted: 6 December 2008.

\* Corresponding author.

2000 *Mathematics Subject Classification.* 46H25.

*Key words and phrases.* Banach algebra, homomorphism, derivation,  $(\varphi, \psi)$ -derivation, weak amenability, second dual.