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## AN EIGENVALUE PROBLEM WITH MIXED BOUNDARY CONDITIONS AND TRACE THEOREMS

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This paper is dedicated to Professor J. E. Pecaric

Submitted by P. K. Sahoo

ABSTRACT. An eigenvalue problem is considered where the eigenvalue appears in the domain and on the boundary. This eigenvalue problem has a spectrum of discrete positive and negative eigenvalues. The smallest positive and the largest negative eigenvalue  $\lambda_{\pm 1}$  can be characterized by a variational principle. We are mainly interested in obtaining non trivial upper bounds for  $\lambda_{-1}$ . We prove some domain monotonicity for certain special shapes using a kind of maximum principle derived by Bandle, v. Bellow and Reichel in [J. Eur. Math. Soc., 10 (2007), 73–104]. We then apply these bounds to the trace inequality.

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