

Banach J. Math. Anal. 2 (2008), no. 2, 42–58

BANACH JOURNAL OF MATHEMATICAL ANALYSIS ISSN: 1735-8787 (electronic) http://www.math-analysis.org

SOME WEIGHTED SUM AND PRODUCT INEQUALITIES IN L^p SPACES AND THEIR APPLICATIONS

R. C. BROWN

This paper is dedicated to Professor Joseph E. Pečarić

Submitted by Th. M. Rassias

ABSTRACT. We survey some old and new results concerning weighted norm inequalities of sum and product form and apply the theory to obtain limitpoint conditions for second order differential operators of Sturm-Liouville form defined in L^p spaces. We also extend results of Anderson and Hinton by giving necessary and sufficient criteria that perturbations of such operators be relatively bounded. Our work is in part a generalization of the classical Hilbert space theory of Sturm-Liouville operators to a Banach space setting.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ALABAMA-TUSCALOOSA, AL 35487-0350, USA

E-mail address: dicbrown@bama.ua.edu

Date: Received: 12 April 2008; Accepted 21 April 2008.

²⁰⁰⁰ Mathematics Subject Classification. Primary: 26D10, 47A30, 34B24; Secondary 47E05. Key words and phrases. Weighted sum inequalities, weighted product inequalities, Sturm Liouville operators, limit-point conditions, relatively bounded perturbations.