# Gauge Bianchi Identities in Higher Order Lagrange Spaces

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***Dedicated to Prof.Dr. Constantin UDRIŞTE on the occasion of his sixtieth birthday***

**Abstract**

In the previous papers ([12] , [13]) we put the bases of a gauge theory which can be applied any time we are dealing with physical phenomena that depend on the coordinates of the k – osculator bundle.

In the above mentioned papers we studied the strength fields of the second order and the Lagrangians generated only by strength fields. Also, for a full Lagrangian $L_0$ we determined the conserved currents and the corresponding conservation laws.

In this paper we shall define (k+1) gauge covariant derivatives of strength fields and we shall determine four types of gauge Bianchi identities.

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**Key words**: gauge transformations, gauge Miron connection, higher order Lagrange spaces, generalized Lagrange metric of order k, gauge Bianchi identities.

## 1 Introduction

The higher order Lagrange spaces constitute an adequate geometrical framework for the development of an integrated gauge theory of the physical fields. Also, the form of interactions between some matter fields can be determined by postulating invariance under a certain group of transformations. In monograph [8] R. Miron gives an original construction of the geometry of higher order Lagrange spaces based on the k – osculator bundle notion. Generalizing some results given in [1], [5], [6], [9] and [14], R. Miron and Gh. Munteanu put the bases of a gauge theory on higher order Lagrange spaces ([8], [10]).

In the paper [12] we studied the strength fields of the second order on the geometrical model given by $GOSC^{(2)}M$. Also, we studied Lagrangians involved gauge fields, defined through strength fields. A full Lagrangian was defined as the sum of the Lagrangian of gauge fields and local gauge invariant Lagrangian of matter fields. For a full Lagrangian $L_0(u)$ we determined in Lagrange manner, the equations of motion. In [13] we studied the local gauge invariance of a full Lagrangian, the conserved currents and the corresponding conservation laws.

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This paper continues the line of [12] and [13]. Using the structure constants of the Lie group G, the gauge fields and an arbitrary gauge Miron connection we shall define (k+1) gauge – covariant derivatives of strength fields and then obtain four types of gauge Bianchi identities.

## 2 Preliminaries

Let M be a real n–dimensional $C^\infty$– differentiable manifold and its k–osculator bundle $(OSC^{(k)}M, \pi, M)$. The local coordinates on the total space $E = OSC^{(k)}M$ are denoted by $u = (x^i, y^{(1)i}, ..., y^{(k)i})$. Let us consider G a compact subgroup in $GL(m, \mathbb{R})$ and $G^{(k)}$ its prolongation of order k. Let $P_G^{(k)}(M)$ be a principal bundle having the base M and structural group $G^{(k)}$. We consider $F = \mathbb{R}^{km}$. A gauge k–osculator bundle $GOSC^{(k)}M$ is a G–structure of order k of the principal bundle $P_G^{(k)}(M)$. The geometrical theory of the gauge $k$–osculator bundle $GOSC^{(k)}M$ is the geometrical theory of the $k$–osculator bundle $OSC^{(k)}M$ restricted to the group $G^{(k)}$.

The notions of gauge transformation in $GOSC^{(k)}M$, nonlinear connection on $GOSC^{(k)}M$, $N$–linear connection on $GOSC^{(k)}M$, generalized Lagrange metric of order $k$, are given in [8] and [10]. The transformations of coordinates on $GOSC^{(k)}M$ are given by

$$(2.1)\qquad \begin{cases} \tilde{x}^i = \tilde{x}^i(x) & \det\left(\dfrac{\partial \tilde{x}^i}{\partial x^j}\right) \neq 0 \\ \tilde{y}^{(1)i} = \dfrac{\partial \tilde{x}^i}{\partial x^j} \cdot y^{(1)j} \\ \dots \\ k\,\tilde{y}^{(k)i} = k \dfrac{\partial \tilde{y}^{(k-1)i}}{\partial y^{(k-1)j}} y^{(k)j} + \dots + \dfrac{\partial \tilde{y}^{(k-1)i}}{\partial x^j} y^{(1)j}. \end{cases}$$

In local coordinates on manifold $GOSC^{(k)}M$, a gauge transformation can be represented by equations of the form

$$(2.2)\qquad \begin{cases} \bar{x}^i = X^i(x) \\ \bar{y}^{(1)i} = Y^{(1)i}(x, y^{(1)}) \\ \dots \\ \bar{y}^{(k)i} = Y^{(k)i}(x, y^{(1)}, ..., y^{(k)}), \end{cases}$$

where

$$\det\left(\frac{\partial X^i}{\partial x^j}\right) \det\left(\frac{\partial Y^{(1)i}}{\partial y^{(1)j}}\right) \cdot \ldots \cdot \det\left(\frac{\partial Y^{(k)i}}{\partial y^{(k)j}}\right) \neq 0.$$

Let a generalized Lagrange metric of order k on $GOSC^{(k)}M$ be given by the components $(g_{ij}(u))$, a gauge nonlinear connection $N = (\underset{(1)}{N}{}^i_j, \underset{(2)}{N}{}^i_j, ..., \underset{(k)}{N}{}^i_j)$, and a gauge Miron connection $D\Gamma N = (L^i_{jk}, \underset{(1)}{C}{}^i_{jk}, ..., \underset{(k)}{C}{}^i_{jk})$ on $GOSC^{(k)}M$ and $L_0(u)$,

$u = \left(x^i, y^{(1)i}, ..., y^{(k)i}\right)$ a Lagrangian defined on the domain $\Omega \subset \mathbb{R}^{(k+1)n}$. Suppose that there exist p differentiable scalar fields (physical fields) $Q^A$, $A = \overline{1,p}$ so that the Lagrangian $L_0$ depends only on the variable $x^i, y^{(1)i}, ..., y^{(k)i}$ by means of $Q^A$ and their derivatives $\dfrac{\delta Q^A}{\delta x^i}$, $\dfrac{\delta Q^A}{\delta y^{(\alpha)i}}$, $\alpha = 1, ..., k$. More accurately, $L_0$ is a scalar field on $GOSC^{(k)}M$ given by

$$L_0\left(x, y^{(1)}, ..., y^{(k)}\right) = L\left(Q^A, \frac{\delta Q^A}{\delta x^i}, \frac{\delta Q^A}{\delta y^{(1)i}}, ..., \frac{\delta Q^A}{\delta y^{(k)i}}\right). \tag{2.3}$$

In order to obtain the gauge–invariant Lagrangians with respect to the local gauge invariance of the Lie group G, we considered a new Lagrangian

$$L_0(u) = L'\left(Q^A, \frac{\delta Q^A}{\delta x^i}, \frac{\delta Q^A}{\delta y^{(\alpha)i}}, H_i^a(u), \overset{(\alpha)^a}{V}_i(u)\right) \tag{2.4}$$

in which $H_i^a(u)$ and $\overset{(\alpha)^a}{V}_i(u)$, $\alpha = 1, ..., k$ are the components of some gauge d–covectors, called local gauge fields, satisfying the following nonhomogeneous conditions of variations

$$\begin{cases} \overset{*}{\delta}\left(H_i^a(u)\right) = \varepsilon^b(u) \cdot f_{bc}^a \cdot H_i^c + \dfrac{\delta \varepsilon^a}{\delta x^i} \\ \overset{*}{\delta}\left(\overset{(\alpha)^a}{V}_i(u)\right) = \varepsilon^b(u) \cdot f_{bc}^a \cdot \overset{(\alpha)^c}{V}_i + \dfrac{\delta \varepsilon^a}{\delta y^{(\alpha)i}}, \end{cases} \tag{2.5}$$

where $f_{bc}^a$ are the structure constants of the Lie group G and $\varepsilon^a(u)$ are differentiable functions.

The study of Lagrangians involving only gauge fields is given in [12]. These Lagrangians can be generated by means of the following functions

$$\overset{(h)^a}{F}_{ij} = \overset{(h)^a}{A}_{ij} - f_{bc}^a \cdot H_i^c \cdot H_j^b + \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}_{ij}^{\ m} \cdot \overset{(\alpha)^a}{V}_m \tag{2.6}$$

$$\overset{(h,v_\alpha)^a}{F}_{ij} = \overset{(h,v_\alpha)^a}{A}_{ij} - f_{bc}^a \cdot H_i^c \cdot \overset{(\alpha)^b}{V}_j + \sum_{\beta=1}^{k} \underset{(\alpha\beta)}{B}{}_{ij}^{\ m} \cdot \overset{(\beta)^a}{V}_m \tag{2.7}$$

$$\overset{(v_\alpha,v_\beta)^a}{F}_{ij} = \overset{(v_\alpha,v_\beta)^a}{A}_{ij} - f_{bc}^a \cdot \overset{(\alpha)^c}{V}_i \cdot \overset{(\beta)^b}{V}_j + \sum_{\gamma=1}^{k} \overset{(\gamma)}{\underset{(\alpha\beta)}{C}}{}_{ij}^{\ m} \cdot \overset{(\gamma)^a}{V}_m, \tag{2.8}$$

where

$$\overset{(h)^a}{A}_{ij} = \frac{\delta H_i^a}{\delta x^j} - \frac{\delta H_j^a}{\delta x^i}, \quad \overset{(h,v_\alpha)^a}{A}_{ij} = \frac{\delta H_i^a}{\delta \overset{(\alpha)^j}{y}} - \frac{\delta \overset{(\alpha)^a}{V}_j}{\delta x^i}, \quad \overset{(v_\alpha,v_\beta)^a}{A}_{ij} = \frac{\delta \overset{(\alpha)^a}{V}_i}{\delta \overset{(\beta)^j}{y}} - \frac{\delta \overset{(\beta)^a}{V}_j}{\delta \overset{(\alpha)^i}{y}}. \tag{2.9}$$

We call $\overset{(h)^a}{F}_{ij}$, $\overset{(h,v_\alpha)^a}{F}_{ij}$ and $\overset{(v_\alpha,v_\beta)^a}{F}_{ij}$ *the horizontal strength fields, the mixed strength fields and* $v_\alpha-$ *vertical strength fields,* respectively.

## 3 Gauge Bianchi identities with respect to covariant derivatives of strength fields

In this section, using the structure constants of the Lie group G, the gauge fields and an arbitrary gauge Miron connection we shall define $(k+1)$ gauge–covariant derivatives of strength fields and then obtain four types of Bianchi identities.

Denote by $K^a_{ij}$ the local components of one of the strength fields $\overset{(h)}{F}{}^a_{ij}$, $\overset{(h,v_\alpha)}{F}{}^a_{ij}$, $\overset{(v_\alpha,v_\beta)}{F}{}^a_{ij}$ given by (2.6), (2.7) and (2.8) respectively. Then we define the *horizontal gauge covariant derivative* of $K^a_{ij}$ as follows

$$K^a_{ij|k} = \frac{\delta K^a_{ij}}{\delta x^k} + f^a_{bc}\cdot K^b_{ij}\cdot H^c_k - K^a_{hj}\cdot L^h_{ik} - K^a_{ih}\cdot L^h_{jk}. \tag{3.1}$$

In a similar way, we define the $v_\alpha-$ *vertical gauge covariant derivative* of $K^a_{ij}$ by

$$K^a_{ij\overset{(\alpha)}{|}k} = \frac{\delta K^a_{ij}}{\delta \overset{(\alpha)}{y}{}^k} + f^a_{bc}\cdot K^b_{ij}\cdot \overset{(\alpha)}{V}{}^c_k - K^a_{hj}\cdot \underset{(\alpha)}{C}{}^h_{ik} - K^a_{ih}\cdot \underset{(\alpha)}{C}{}^h_{jk}. \tag{3.2}$$

By a direct but very long calculation we can prove the following three results

**Proposition 3.1.** *Both covariant derivatives define d–gauge tensor fields of type (0,3). More precisely, with respect to (2.1) and (2.2) we have*

$$K^a_{ij|k} = \sigma^h_i\cdot\sigma^l_j\cdot\sigma^r_k\cdot \tilde{K}^a_{hl|r} \tag{3.3}$$

$$K^a_{ij\overset{(\alpha)}{|}k} = \sigma^h_i\cdot\sigma^l_j\cdot\sigma^r_k\cdot \tilde{K}^a_{hl\overset{(\alpha)}{|}r}, \tag{3.4}$$

*and respectively*

$$K^a_{ij|k} = X^h_i\cdot X^l_j\cdot X^r_k\cdot \bar{K}^a_{hl|r} \tag{3.5}$$

$$K^a_{ij\overset{(\alpha)}{|}k} = X^h_i\cdot X^l_j\cdot X^r_k\cdot \bar{K}^a_{hl\overset{(\alpha)}{|}r}, \tag{3.6}$$

where $\sigma^j_i = \dfrac{\partial \tilde{x}^j}{\partial x^i}$ and $X^h_i = \dfrac{\partial X^h}{\partial x^i}$.

**Proposition 3.2.** *With respect to the local gauge action of Lie group G, the gauge covariant derivatives verify the following homogeneous laws of transformation*:

$$\overset{*}{\delta}\left(K^a_{ij|k}\right) = \varepsilon^b\cdot f^a_{bc}\cdot K^c_{ij|k} \tag{3.7}$$

$$\overset{*}{\delta}\left(K^a_{ij\overset{(\alpha)}{|}k}\right) = \varepsilon^b\cdot f^a_{bc}\cdot K^c_{ij\overset{(\alpha)}{|}k}. \tag{3.8}$$

**Proposition 3.3.** *With respect to d–vector fields of adapted basis of* $T_uGOSC^{(k)}M$, *the following Jacobi identities hold*:

$$\sum_{(i,j,k)}\left(\frac{\delta \underset{(0\alpha)}{R}{}^{m}_{ij}}{\delta x^k}+\sum_{\beta=1}^{k}\underset{(0\beta)}{R}{}^{h}_{ij}\cdot\underset{(\beta\alpha)}{B}{}^{m}_{kh}\right)=0 \tag{3.9}$$

$$\begin{aligned}&\sum_{\beta=1}^{k}\left(\underset{(0\beta)}{R}{}^{h}_{ij}\cdot\overset{(\gamma)}{\underset{(\beta\alpha)}{C}}{}^{m}_{hk}-\underset{(\alpha\beta)}{B}{}^{p}_{jk}\cdot\underset{(\beta\gamma)}{B}{}^{m}_{ip}+\underset{(\alpha\beta)}{B}{}^{p}_{ik}\cdot\underset{(\beta\gamma)}{B}{}^{m}_{jp}\right)=\\&=\frac{\delta\underset{(0\gamma)}{R}{}^{m}_{ij}}{\delta \overset{(\alpha)}{y}{}^{k}}+\frac{\delta\underset{(\alpha\gamma)}{B}{}^{m}_{jk}}{\delta x^i}-\frac{\delta\underset{(\alpha\gamma)}{B}{}^{m}_{ik}}{\delta x^j}\end{aligned} \tag{3.10}$$

$$\begin{aligned}&\sum_{\gamma=1}^{k}\left(\underset{(\alpha\gamma)}{B}{}^{p}_{ij}\cdot\overset{(\zeta)}{\underset{(\gamma\beta)}{C}}{}^{m}_{pk}-\underset{(\beta\gamma)}{B}{}^{p}_{ik}\cdot\overset{(\zeta)}{\underset{(\gamma\alpha)}{C}}{}^{m}_{pj}-\underset{(\gamma\zeta)}{B}{}^{m}_{ip}\cdot\overset{(\gamma)}{\underset{(\alpha\beta)}{C}}{}^{p}_{jk}\right)\\&=\frac{\delta\underset{(\alpha\zeta)}{B}{}^{m}_{ij}}{\delta\overset{(\beta)}{y}{}^{k}}-\frac{\delta\underset{(\beta\zeta)}{B}{}^{m}_{ik}}{\delta\overset{(\alpha)}{y}{}^{j}}+\frac{\delta\overset{(\zeta)}{\underset{(\alpha\beta)}{C}}{}^{m}_{jk}}{\delta x^i}\end{aligned} \tag{3.11}$$

$$\begin{aligned}&\sum_{\zeta=1}^{k}\left(\overset{(\zeta)}{\underset{(\alpha\beta)}{C}}{}^{p}_{ij}\cdot\overset{(\eta)}{\underset{(\zeta\gamma)}{C}}{}^{m}_{pk}+\overset{(\zeta)}{\underset{(\beta\gamma)}{C}}{}^{p}_{jk}\cdot\overset{(\eta)}{\underset{(\zeta\alpha)}{C}}{}^{m}_{pi}+\overset{(\zeta)}{\underset{(\gamma\alpha)}{C}}{}^{p}_{ki}\cdot\overset{(\eta)}{\underset{(\zeta\beta)}{C}}{}^{m}_{pj}\right)\\&=\frac{\delta\overset{(\eta)}{\underset{(\alpha\beta)}{C}}{}^{m}_{ij}}{\delta\overset{(\gamma)}{y}{}^{k}}+\frac{\delta\overset{(\eta)}{\underset{(\beta\gamma)}{C}}{}^{m}_{jk}}{\delta\overset{(\alpha)}{y}{}^{i}}+\frac{\delta\overset{(\eta)}{\underset{(\gamma\alpha)}{C}}{}^{m}_{ki}}{\delta\overset{(\beta)}{y}{}^{j}}.\end{aligned} \tag{3.12}$$

Here and in sequel, by $\sum\limits_{(i,j,k)}$ we mean the cyclic sum with respect to $(i,j,k)$. The above propositions are useful for proving the following main results

**Theorem 3.1.** *The following gauge Bianchi identity with respect to the horizontal gauge–covariant derivative of the horizontal strength fields holds*:

$$\sum_{(i,j,k)}\left\{\overset{(h)}{F}{}^{a}_{ij|k}+\sum_{\alpha=1}^{k}\underset{(0\alpha)}{R}{}^{m}_{ij}\cdot\overset{(h,v_\alpha)}{F}{}^{a}_{km}+\underset{(00)}{T}{}^{h}_{ij}\cdot\overset{(h)}{F}{}^{a}_{kh}\right\}=0. \tag{3.13}$$

*Proof.* First, we have

$$\begin{aligned}\frac{\delta\overset{(h)}{F}{}^{a}_{ij}}{\delta x^k}&=\frac{\delta^2H^a_i}{\delta x^k\delta x^j}-\frac{\delta^2H^a_j}{\delta x^k\delta x^i}-f^a_{bc}\cdot\left(\frac{\delta H^c_i}{\delta x^k}\cdot H^b_j+\frac{\delta H^b_j}{\delta x^k}\cdot H^c_i\right)+\\&+\sum_{\alpha=1}^{k}\frac{\delta\underset{(0\alpha)}{R}{}^{m}_{ij}}{\delta x^k}\cdot\overset{(\alpha)}{V}{}^{a}_{m}+\sum_{\alpha=1}^{k}\underset{(0\alpha)}{R}{}^{m}_{ij}\cdot\frac{\delta\overset{(\alpha)}{V}{}^{a}_{m}}{\delta x^k}.\end{aligned} \tag{3.14}$$

Using (3.14) we can easily obtain

$$\sum_{(i,j,k)}\left(\frac{\delta \overset{(h)^a}{F}_{ij}}{\delta x^k} - f^a_{bc}\cdot H^b_i \cdot \overset{(h)^c}{A}_{jk} + \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}^{m}_{ij}\cdot\left(\frac{\delta H^a_k}{\delta \overset{(\alpha)^m}{y}} - \frac{\delta \overset{(\alpha)^a}{V}_m}{\delta x^k}\right)\right.$$
$$\left. - \sum_{\alpha=1}^{k}\frac{\delta \underset{(0\alpha)}{R}{}^{m}_{ij}}{\delta x^k}\cdot \overset{(\alpha)^a}{V}_m\right) = 0. \tag{3.15}$$

Now, using (3.9) and (3.15) we have

$$\sum_{(i,j,k)}\left(\frac{\delta \overset{(h)^a}{F}_{ij}}{\delta x^k} - f^a_{bc}\cdot H^b_i \cdot \overset{(h)^c}{A}_{jk}\right.$$
$$\left. + \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}^{m}_{ij}\cdot\left(\frac{\delta H^a_k}{\delta \overset{(\alpha)^m}{y}} - \frac{\delta \overset{(\alpha)^a}{V}_m}{\delta x^k} + \sum_{\beta=1}^{k} \underset{(\alpha\beta)}{B}{}^{h}_{km}\cdot \overset{(\beta)^a}{V}_h\right)\right) = 0. \tag{3.16}$$

Taking into account (2.6) and the Jacobi identity from the general theory of Lie groups ($f^a_{bc}\cdot f^c_{de} + f^a_{dc}\cdot f^c_{eb} + f^a_{ec}\cdot f^c_{bd} = 0$) we obtain the following relation

$$f^a_{bc}\cdot \sum_{(i,j,k)}\left\{\left(\overset{(h)^b}{F}_{ij} - \overset{(h)^b}{A}_{ij} - \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}^{m}_{ij}\cdot \overset{(\alpha)^b}{V}_m\right)\cdot H^c_k\right\} = 0. \tag{3.17}$$

From (3.16) and (3.17) we have

$$\sum_{(i,j,k)}\left\{\frac{\delta \overset{(h)^a}{F}_{ij}}{\delta x^k} + f^a_{bc}\cdot \overset{(h)^b}{F}_{ij}\cdot H^c_k + \right.$$
$$\left. + \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}^{m}_{ij}\cdot\left(\frac{\delta H^a_k}{\delta \overset{(\alpha)^m}{y}} - \frac{\delta \overset{(\alpha)^a}{V}_m}{\delta x^k} + k\sum_{\beta=1} \underset{(\alpha\beta)}{B}{}^{h}_{km}\cdot \overset{(\beta)^a}{V}_h - f^a_{bc}\cdot H^c_k \cdot \overset{(\alpha)^b}{V}_m\right)\right\} = 0 \Longleftrightarrow$$

$$\sum_{(i,j,k)}\left\{\overset{(h)^a}{F}_{ij|k} + \overset{(h)^a}{F}_{hj}\cdot L^h_{ik} + \overset{(h)^a}{F}_{ih}\cdot L^h_{jk} + \sum_{\alpha=1}^{k} \underset{(0\alpha)}{R}{}^{m}_{ij}\cdot \overset{(h,v_\alpha)^a}{F}_{km}\right\} = 0. \tag{3.18}$$

Because $\underset{(00)}{T}{}^{h}_{ij} = L^h_{ij} - L^h_{ji}$ we have

$$\sum_{(i,j,k)}\left(\overset{(h)^a}{F}_{hj}\cdot L^h_{ik} + \overset{(h)^a}{F}_{ih}\cdot L^h_{jk}\right) = \sum_{(i,j,k)} \overset{(h)^a}{F}_{kh}\cdot \underset{(00)}{T}{}^{h}_{ij}. \tag{3.19}$$

From (3.18) and (3.19) we can easily obtain the conclusion of theorem. □

Next, by $\sum\limits_{\substack{(i,j,k)\\(\alpha,\beta,\gamma)}}$ we mean the cyclic sum with respect to $(i,j,k)$ and $(\alpha,\beta,\gamma)$ when we move simultaneously the indices of these triples. In the same manner we can prove the following two results

**Theorem 3.2.** *The following gauge Bianchi identity with respect to the $v_\alpha$–gauge–covariant derivatives of the $v_\beta$–vertical strength fields holds:*

$$\sum_{\substack{(i,j,k)\\(\alpha,\beta,\gamma)}}\left\{\overset{(v_\alpha,v_\beta)^a}{F}{}_{ij\overset{(\gamma)}{|}k}+\sum_{\varsigma=1}^{k}\overset{(\varsigma)}{\underset{(\alpha\beta)}{C}}{}^{m}_{ij}\cdot\overset{(v_\gamma,v_\varsigma)^a}{F}{}_{km}+\underset{(\gamma)}{D}{}^{hp}_{ijk}\cdot\overset{(v_\alpha,v_\beta)^a}{F}{}_{hp}\right\}=0, \tag{3.20}$$

*where*

$$\underset{(\gamma)}{D}{}^{hp}_{ijk}=\underset{(\gamma)}{C}{}^{h}_{ik}\cdot\delta^p_j+\underset{(\gamma)}{C}{}^{p}_{jk}\cdot\delta^h_i. \tag{3.21}$$

**Theorem 3.3.** *The following mixed gauge Bianchi identities with respect to the gauge $v_\alpha$– covariant derivatives of the mixed strength fields, and the horizontal covariant derivative of $v_\beta$–vertical strength fields, respectively hold:*

$$\begin{aligned}&\overset{(h,v_\alpha)^a}{F}{}_{ij\overset{(\beta)}{|}k}-\overset{(h,v_\beta)^a}{F}{}_{ik\overset{(\alpha)}{|}j}+\overset{(v_\alpha,v_\beta)^a}{F}{}_{jk|i}+\sum_{\gamma=1}^{k}\{\overset{(\gamma)}{\underset{(\alpha\beta)}{C}}{}^{m}_{jk}\cdot\overset{(h,v_\gamma)^a}{F}{}_{im}+\\&+\underset{(\beta\gamma)}{B}{}^{m}_{ik}\cdot\overset{(v_\gamma,v_\alpha)^a}{F}{}_{mj}-\underset{(\alpha\gamma)}{B}{}^{m}_{ij}\cdot\overset{(v_\gamma,v_\beta)^a}{F}{}_{mk}\}+\underset{(\beta)}{D}{}^{hp}_{ijk}\cdot\overset{(h,v_\alpha)^a}{F}{}_{hp}-\\&-\underset{(\alpha)}{D}{}^{hp}_{ikj}\cdot\overset{(h,v_\beta)^a}{F}{}_{hp}+E^{hp}_{jki}\cdot\overset{(v_\alpha,v_\beta)^a}{F}{}_{hp}=0\end{aligned} \tag{3.22}$$

$$\begin{aligned}&\overset{(h,v_\alpha)^a}{F}{}_{ij|k}-\overset{(h,v_\alpha)^a}{F}{}_{kj|i}+\overset{(h)^a}{F}{}_{ki\overset{(\alpha)}{|}j}+\\&+\overset{(h,v_\alpha)^a}{F}{}_{hj}\cdot\underset{(00)}{T}{}^{h}_{ik}+\overset{(h,v_\alpha)^a}{F}{}_{ph}\cdot F^{hp}_{jki}+\overset{(h)^a}{F}{}_{hp}\cdot\underset{(\alpha)}{D}{}^{hp}_{kij}+\\&+\sum_{\beta=1}^{k}\left\{\underset{(\alpha\beta)}{B}{}^{m}_{ij}\cdot\overset{(h,v_\beta)^a}{F}{}_{km}-\underset{(\alpha\beta)}{B}{}^{m}_{kj}\cdot\overset{(h,v_\beta)^a}{F}{}_{im}-\underset{(0\beta)}{R}{}^{m}_{ik}\cdot\overset{(v_\alpha,v_\beta)^a}{F}{}_{jm}\right\}=0,\end{aligned} \tag{3.23}$$

*where*

$$E^{hp}_{jki}=L^h_{ji}\cdot\delta^p_k+L^p_{ki}\cdot\delta^h_j \tag{3.24}$$

$$F^{hp}_{jki}=L^h_{jk}\cdot\delta^p_i-L^h_{ji}\cdot\delta^p_k, \tag{3.25}$$

*and the functions $\underset{(\alpha)}{D}{}^{hp}_{kij}$ are given by (3.21).*

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