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Transfer Function Model Identification

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Abstract

Transfer function models are widely used in engineering and in economics. In this article, an automatic procedure is proposed to identify such models. The proposed procedure can directly handle nonstationarity, outliers and other deterministic effects such as Trading Day or Easter. The procedure is applied to one simulated series and one real series.

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1. Introduction

In economics and other disciplines investigators often employ transfer function models. In its simplest form, a transfer function model can be written as

$$y_t = C + \nu(B)x_t + n_t, \tag{1.1}$$

where y_t is the output series or endogenous variable, x_t is the input series or exogenous variable, n_t is the disturbance series that is uncorrelated with x_t, $\nu(z) = \sum_{i=0}^{\infty} \nu_i z^i$ is a filter, usually rational, that is applied to the input x_t, B is the backshift operator, $By_t = y_{t-1}$, and C is a constant. For example, when x_t is a leading indicator, an equation like (1.1) is often used by economists either to describe the relationship between y_t and x_t, or to improve the forecasting performance of y_t, or both. The improvement in forecasting is particularly relevant if the turning points of y_t can be anticipated from those of x_t.

The input variable, x_t, in (1.1) is assumed to be strongly exogenous (Harvey, 1989, pp. 374–375). This means that x_t can be treated as fixed and the parameters in (1.1) can be estimated independently of the parameters in the model followed by $\{x_t\}$ if $\{x_t\}$ is stochastic. Thus, even if $\{x_t\}$ is stochastic and follows a well specified model, the unknown parameters contained in the initial conditions to obtain the filtered series $z_t = \nu(B)x_t$ must be estimated using the model (1.1) and not the model followed by $\{x_t\}$.

In this article, a new automatic procedure is proposed to identify transfer function models. This procedure has been in use at the Ministry of Economics and Finance of Spain for forecasting purposes for some time. It has been programmed by the author in MATLAB and the results so far are satisfactory.

The outline of the article is as follows. Section 2 briefly reviews transfer function models. Section 3 describes the proposed automatic procedure to identify transfer function models. In Section 4 we illustrate the proposed procedure with one simulated and one real example.

2. Brief Review of Identifying Methods for Transfer Function Models

In this section we briefly discuss some of the most widely used methods to identify transfer function models. These are the prewhitening method proposed by Box and Jenkins (Box et al., 1994), the linear transfer function (LTF) method proposed by Liu and Hanssens (1982), and the procedure proposed by Tsay (1985).

Assuming an output variable, y_t, and m input variables, $x_{1t}, \ldots, x_{mt}$, a transfer function model can be written as

$$y_t = C + \frac{\omega_1(B)}{\delta_1(B)}x_{1t} + \frac{\omega_2(B)}{\delta_2(B)}x_{2t} + \cdots + \frac{\omega_m(B)}{\delta_m(B)}x_{mt} + \frac{\theta(B)}{\phi(B)}a_t,$$

where B is the backshift operator, $By_t = y_{t-1}$,

$$\begin{aligned}
\omega_i(B) &= (\omega_{i0} + \omega_{i1}B + \omega_{i2}B^2 + \cdots + \omega_{ih_i}B^{h_i})B^{b_i} \\
\delta_i(B) &= 1 + \delta_{i1}B + \cdots + \delta_{ir_i}B^{r_i} \\
\phi(B) &= 1 + \phi_1 B + \cdots + \phi_p B^p \\
\theta(B) &= 1 + \theta_1 B + \cdots + \theta_q B^q,
\end{aligned}$$

$\{a_t\}$ is white noise, usually assumed to be i.i.d. and Gaussian with zero mean. In addition, $\{a_t\}$ and the $\{x_{it}\}$ are assumed to be mutually and serially uncorrelated. The polynomials $\phi(z)$ and $\theta(z)$ can have multiplicative form in case seasonality is present.

The prewhitening method to identify transfer function models is described in Box et al. (1994). This method has several drawbacks. For this reason, we will consider in this article the identification method proposed by Liu and Hanssens (1982), known as linear transfer function (LTF), and also the procedure proposed by Tsay (1985).

The LTF method is based on the following ideas. To simplify the notation, suppose only one input in the transfer function equation and denote by $\nu(z) =$

$\omega(z)/\delta(z)$ its filter. Then, we can consider the approximation

$$\nu(z) = \nu_0 + \nu_1 z + \nu_2 z^2 + \cdots ,$$

and we can try to estimate the weights $\{\nu_j\}$ first. The whole procedure is as follows:

1. Estimate the weights $\{\nu_j\}$ assuming some model for N_t in the transfer function equation, $y_t = \nu(z)x_t + N_t$. The model for N_t is usually an AR(1) or, if there is seasonality, an $AR(1) \times AR(s)$, where s is the number of seasons.

2. Identify a model for $\{N_t\}$.

3. Identify the polynomials $\omega(z)$ and $\delta(z)$ for the best approximation $\omega(z)/\delta(z) \simeq \nu(z)$.

In practice, a finite approximation for the filter $\nu(z)$ is used, so that a model of the form

$$y_t = C + (\nu_0 + \nu_1 B + \nu_2 B^2 + \cdots + \nu_k B^k)x_t + N_t \tag{2.1}$$

is considered. After steps 1 and 2, a generalization of the corner method (Beguin et al. 1980) is used to identify the polynomials $\omega(z)$ and $\delta(z)$ such that $\omega(z)/\delta(z) \simeq \nu(z)$.

Tsay (1985) proposed as a first step in transfer function identification to fit an autoregressive vector model to the random vector formed with the output and all of the inputs. In this way, a test of unidirectional causality can be implemented, the number of lags in the approximation for the input filters can be determined, and the weights of the approximation can be estimated.

Based on the identification and estimation of this autoregressive model, Tsay (1985) proposed a method to identify the output model and filters for the inputs. These last filters are also identified using the corner method.

3. An Automatic Procedure to Identify Transfer Function Models

To describe the procedure, suppose for simplicity that there is only one input. Then, in the first stage of the procedure, we also use a model of the form (2.1) to estimate the weights, $\{\nu_i\}$, $i = 1, \ldots, k$. However, instead of using an AR model for N_t, we use an automatic ARIMA identification procedure to identify an ARIMA model for N_t. The automatic procedure is similar to the one used in the program TRAMO (Gómez and Maravall, 1992) and has been programmed by the author. This allows us to identify and estimate some deterministic effects as well, like Easter effect, trading day effect, outliers, etc. In this first stage, the number of the first insignificant ν_i parameters is equal to the time delay parameter, b, so that

$$\omega(B) = (\omega_0 + \omega_1 B + \omega_2 B^2 + \cdots + \omega_h B^h)B^b.$$

In the second stage, we first reestimate the model without the first b insignificant ν_i weights. Then, using the newly estimated weights, $\hat{\nu}_i$, $i = 0, 1, 2, \ldots, k$, we use the method proposed by Schank (Schank, 1967) to estimate the coefficients ω_i and δ_i in

$$\nu(z) = \frac{\omega_0 + \omega_1 z + \omega_2 z^2 + \cdots + \omega_h z^h}{1 + \delta_1 z + \cdots + \delta_r z^r} \tag{3.1}$$

for several choices of h and r. More specifically, equating coefficients in (3.1) implies

$$\left[\begin{array}{ccccc} \nu_0 & & & & \\ \nu_1 & \nu_0 & & & \\ \nu_2 & \nu_1 & \nu_0 & & \\ \vdots & \vdots & \vdots & \ddots & \\ \nu_n & \nu_{n-1} & \nu_{n-2} & \cdots & \nu_0 \\ \hline \nu_{n+1} & \nu_n & \nu_{n-1} & \cdots & \nu_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \nu_k & \nu_{k-1} & \nu_{k-2} & \cdots & \nu_{k-n} \end{array}\right] \left[\begin{array}{c} 1 \\ \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{array}\right] = \left[\begin{array}{c} \omega_0 \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \\ \hline 0 \\ \vdots \\ 0 \end{array}\right],$$

where $n = \max\{h, r\}$, $\omega_i = 0$ if $i > h$ and $\nu_i = 0$ if $i > r$. To solve the previous system, we first estimate the δ_i coefficients by ordinary least squares using the last part of the system, that is, those equations that have a zero on the right hand side. Then, replacing in (3.1) the δ_i coefficients with the estimated ones, $\hat{\delta}_i$, we set up a new system of linear equations to estimate the ω_i coefficients. We could use the first $n+1$ equations of the previous system to estimate the ω_i coefficients. However, it would be desirable to use all the information contained in the sample. This is done by expanding first

$$1/(1 + \hat{\delta}_1 z + \cdots + \hat{\delta}_r z^r) = 1 + \gamma_1 z + \gamma_1 z^2 + \cdots$$

and then equating coefficients in

$$\nu(z) = (\omega_0 + \omega_1 z + \omega_2 z^2 + \cdots + \omega_h z^h)(1 + \gamma_1 z + \cdots)$$

until we get $k+1$ equations. That is, we solve the linear system in the least squares sense

$$\left[\begin{array}{ccccc} 1 & & & & \\ \gamma_1 & 1 & & & \\ \gamma_2 & \gamma_1 & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ \gamma_n & \gamma_{n-1} & \gamma_{n-2} & \cdots & 1 \\ \hline \gamma_{n+1} & \gamma_n & \gamma_{n-1} & \cdots & \gamma_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_k & \gamma_{k-1} & \gamma_{k-2} & \cdots & \gamma_{k-n} \end{array}\right] \left[\begin{array}{c} \omega_0 \\ \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{array}\right] = \left[\begin{array}{c} \nu_0 \\ \nu_1 \\ \nu_2 \\ \vdots \\ \nu_n \\ \hline \nu_{n+1} \\ \vdots \\ \nu_k \end{array}\right].$$

Since the ω_i and δ_i coefficients are estimated by ordinary least squares in Schank's method, we can identify the optimum h and r using some information criterion, like AIC or BIC. More specifically, assuming $0 \leq h, r, \leq 2$, we compute ω_i and δ_i for all possible combinations of h and r and, for each combination, we compute the errors $e_i = \hat{\nu_i} - \tilde{\nu_i}$, $i = 1, 2, \ldots, k$, where $\hat{\nu_i}$ and $\tilde{\nu_i}$ are the weights obtained in the first stage of the procedure and the weights computed with the ω_i and δ_i coefficients estimated for that combination, respectively. The criterion is of the form $C_{h,r} = \ln(\hat{\sigma}^2) + C(k)(h+1+r)$, where $\hat{\sigma}^2 = (1/k)\sum_{i=1}^{k} e_i^2$ and $C(k)$ is some penalty term. We select h and r that minimize $C_{h,r}$. In the proposed procedure, we use Corrected AIC as criterion because it seems to work better than AIC or BIC in small samples.

In the third stage, we estimate the transfer function model identified in the previous two stages using maximum likelihood.

4. Examples

To illustrate the procedure, we first use a simulated series that follows the model

$$(1 - B)Y_t = (3 - 2B)(1 - B)X_{t-1} + (1 - .7B)a_t,$$

where B is the backshift operator, $By_t = y_{t-1}$, and has 130 observations. The series is the series tf2, included in a collection of time series for research and teaching that is distributed with the computer package SCA.

The automatic procedure correctly identifies both, the model for N_t and the filter for the input. The estimated model is

$$(1 - B)Y_t = (2.97 - 1.97B)(1 - B)X_{t-1} + (1 - .6B)a_t.$$

In the second example, the aim is to forecast the Spanish Consumer Price Index (SCPI), base year 2001, using two exogenous inputs that are believed to have some information about SCPI. These are the Spanish Import Price Index for Consumer Goods (base year 2000) and the Spot Prices of Crude Oil in pesetas of the Brent barrel. This last series is considered as deterministic and its forecasts are the corresponding Futures Market Prices.

We fit a transfer function model to the logs of SCPI for the sample 1993:1–2006:3. We use twelve lags of each input series in the first stage of the proposed procedure. No delay parameter is identified for any of the inputs and the identified model for N_t is $(0,1,0)(1,1,0)_{12}$. This last model is

$$\begin{aligned}\nabla\nabla_{12}y_t &= (0.18+0.15B)\nabla\nabla_{12}x_{1t} + \frac{0.01}{1-0.90B+0.45B^2}\nabla\nabla_{12}x_{2t} \\ &\quad + \frac{1}{1+0.41B^{12}}a_t,\end{aligned}$$

where x_{1t} and x_{2t} are the Spanish Import Price Index for Consumer Goods and the Spot Prices of Crude Oil, respectively. The fit is good and there are no signs of model inadequacy.

References

[1] Beguin, J. M., Gourieroux, C. and Monfort, A. (1980), "Identification of a Mixed Autoregressive–Moving Average Process: the Corner Method", in *Time Series* (Anderson, O. D., ed.), Amsterdam: North–Holland.

[2] Box, G.E.P., Jenkins, G.M., and Reinsel, G. C. (1994), *Time Series Analysis, Forecasting and Control*, New Jersey: Prentice Hall.

[3] Gómez, V. and Maravall, A. (1997), "Programs TRAMO and SEATS, Instructions for the User (Beta Version: June 1997)", Working Paper N. 97001, Dirección General De Presupuestos, Ministry of Finance, Madrid, Spain.

[4] Harvey, A. C. (1989), *Forecasting, Structural Models and the Kalman Filter*, Cambridge, U.K.: Cambridge University Press.

[5] Liu, L. M., and Hanssens, D. M. (1982), "Identification of Multiple–Input Transfer Function Models", *Communications in Statistics, Theory and Methods*, **11**, 297–314.

[6] Schank, J. L. (1967), "Recursion Filters for digital Processing", *Geophysics*. **32**, 32–51

[7] Tsay, R. S. (1985), "Model Identification in Dynamic Regression (Distributed Lag) Models", *Journal of Business and Economic Statistics*, **3**, 228–237.

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Víctor Gómez is technical advisor at the Ministry of Economics and Finance of Spain. He has published several research articles in the Journal of American Statistics, the Journal of Business and Economic Statistics, the Journal of Time

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