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ON MARCZEWSKI SETS AND SOME IDEALS

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Abstract. Using the methods of Brown and Walsh, we get condition guaranteeing that, for an ideal $\mathcal{I}$ of sets in a perfect Polish space some (s^0) sets are not in $\mathcal{I}$. A few examples and corollaries are given.

0. Introduction

Papers [**Br**], [**W1**], [**W2**] and [**C**] made a significant progress in the studying of (s^0) sets introduced by Marczewski in [**Sz**]. One of the main results states that there exists a nonmeasurable (s^0) set without the Baire property. That was proved in [**Br**] under CH and in [**W1**], [**W2**], [**C**] within ZFC. We analyse the schemes from [**Br**] and [**W1**], [**W2**] and get two criteria for an ideal $\mathcal{I}$ (of sets in a perfect Polish space X) to satisfy $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}_0$ denotes the ideal of all (s^0) sets (in X). The original proofs we base on need only a slight modification. However, we give new versions in full. We describe some applications.

Throughout the paper, we fix a perfect Polish space X. A set which has no perfect subset is called totally imperfect. A set $E \subseteq X$ is called an (s^0) set if each perfect set has a perfect subset disjoint from E (see [**Sz**]). Obviously, (s^0) sets are totally imperfect and, moreover, they form an ideal (see [**Sz**]) which will be written as $\mathcal{I}_0$.

For any ideal $\mathcal{I} \subseteq \mathcal{P}(X)$, we always assume that $X \notin \mathcal{I}$ (here $\mathcal{P}(X)$ is the power set of X). The cardinality of continuum is denoted by c.

Further, the following lemma will be useful.

0.1. Lemma. *Let $A \subseteq X$ be an uncountable analytic set and $E \subseteq X$. If $|A \cap E| < c$, then there exists a perfect set $P \subseteq A$ missing E.*

Proof. Find a perfect $P \subseteq A$ (cf. [**Kr,** §39.I]) and c pairwise disjoint perfect subsets of P. At least one of them misses E. $\square$

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1. The First Criterion

A family $\tilde{\mathcal{I}}$ is called a base of an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ if $\tilde{\mathcal{I}} \subseteq \mathcal{I}$ and, for each $A \in \mathcal{I}$, there exists $B \in \tilde{\mathcal{I}}$ containing A. We denote

$$\operatorname{cof}(\mathcal{I}) = \min\{\, |\tilde{\mathcal{I}}| : \tilde{\mathcal{I}} \text{is base of } \mathcal{I}\,\}\,,$$
$$\operatorname{cov}(\mathcal{I}) = \min\{\, |\mathcal{H}| : \mathcal{H} \subseteq \mathcal{I} \text{ and } \cup\mathcal{H} = X\,\}\,.$$

It is evident that $\operatorname{cov}(\mathcal{I}) \le \operatorname{cof}(\mathcal{I})$.

We say that $\mathcal{I}$ has property (P) if each perfect set in X has a perfect set belonging to $\mathcal{I}$ (cf. [**Ba1**]).

The following proposition generalizing the method from Example 3 in [**Br**] has been inspired by some comment contained in [**C**].

1.1. Proposition (Criterion 1). *Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal such that*

(a) $\operatorname{cov}(\mathcal{I}) = \operatorname{cof}(\mathcal{I}) = c$,

(b) *$\mathcal{I}$ has property* (P).

Then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$.

Proof. Since $\operatorname{cof}(\mathcal{I}) = c$, there is a base $\tilde{\mathcal{I}}$ of $\mathcal{I}$ with $|\tilde{\mathcal{I}}| = c$. Let $\{A_\alpha : \alpha < c\}$ be an enumeration of sets A such that $X \setminus A \in \tilde{\mathcal{I}}$. Let $\{P_\alpha : \alpha < c\}$ be an enumeration of all perfect subsets of X. By virtue of (b), choose a perfect $Q_0 \in \mathcal{I}$ contained in P_0. Pick any x_0 in $A_0 \setminus Q_0$. If $0 < \alpha < c$ and if x_β, for $\beta < \alpha$, are defined, choose a perfect subset $Q_\alpha \in \mathcal{I}$ of P_α and let

$$\mathcal{F}_\alpha = \{Q_\beta : \beta \le \alpha\} \cup \{x_\beta : \beta < \alpha\}\,.$$

Observe that $A_\alpha \setminus \cup\mathcal{F}_\alpha \notin \mathcal{I}$. Indeed, if it is not the case, then for

$$\mathcal{H} = \mathcal{F}_\alpha \cup \{A_\alpha \setminus \cup\mathcal{F}_\alpha\} \cup \{X \setminus A_\alpha\}\,,$$

we would get $|\mathcal{H}| < c$, $\cup\mathcal{H} = X$, which contradicts $\operatorname{cov}(\mathcal{I}) = c$. Now, pick any x_α in $A_\alpha \setminus \cup\mathcal{F}_\alpha$. If the induction is finished, set $E = \{x_\alpha : \alpha < c\}$.

To show that E is an (s^0) set, consider any P_α. By the construction, $Q_\alpha \cap E \subseteq \{x_\beta : \beta < \alpha\}$. So, by Lemma 0.1, there is a perfect subset of Q_α (thus of P_α) which misses E.

To show $E \notin \mathcal{I}$, suppose that $E \in \mathcal{I}$ and choose $\tilde{E} \in \tilde{\mathcal{I}}$ containing E. Then $X \setminus \tilde{E} = A_\alpha$ for some $\alpha < c$. We have $A_\alpha \subseteq X \setminus E$, which implies $x_\alpha \notin E$, a contradiction. $\square$

Since $\operatorname{cov}(\mathcal{I}) \le \operatorname{cof}(\mathcal{I})$, it is suffices to assume in (a) that $c \le \operatorname{cov}(\mathcal{I})$ and $\operatorname{cof}(\mathcal{I}) \le c$. Property (P) seems rather strong. Note that it is not necessary for $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$. Indeed, if $\mathcal{I}$ is an ideal with property (P) and $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$, then throwing out all perfect subsets of a fixed perfect set from $\mathcal{I}$, we get the ideal $\mathcal{I}^*$ for which

(P) fails to hold and $\mathcal{I}_0 \setminus \mathcal{I}^* \neq \emptyset$. In Section 2 we give examples of quite large and regular ideals without property (P) which do not contain all (s^0) sets.

Coming back to the sources of Criterion 1; i.e. to Example 3 from [**Br**], consider the case when $\mathcal{I}$ is the ideal $\mathcal{L}$ of the Lebesgue null sets in the real line $\mathbb{R}$. It is obvious that $\mathrm{cof}\,(\mathcal{L}) \le c$, and that $\mathcal{L}$ has property (P). The statement $\mathrm{cov}\,(\mathcal{L}) = c$ is implied by CH (or by MA) but is not equivalent. By Criterion 1, we get $\mathcal{I}_0 \setminus \mathcal{L} \neq \emptyset$. This easily implies the existence of an (s^0) set which is nonmeasurable (cf. Corollary 3.2). That was obtained in [**W2**] within ZFC. The first step of the proof is Theorem 2.2 from [**W1**] (for a generalization, see Section 2 of our paper). The second step uses the Fubini theorem. The same technique repeats when the measure is replaced by category (for other cases, see [**Ba2**]. However, if we have no analogues of the Fubini theorem, it can be unclear how to continue the first step. Thus Criterion 1 may help.

1.2. Example. Consider an infinite $K \subseteq \omega = \{0, 1, 2, \dots\}$ and a set $E \subseteq 2^\omega$ where 2^ω is the Cantor space of all infinite sequences with terms from $\{0, 1\}$. Let $\Gamma(E, K)$ be the following game between two players I and II. They choose consecutive terms of a sequence $x = \langle x(0), x(1), \dots \rangle \in 2^\omega$. Player I picks $x(i)$ if $i \notin K$ and Player II — if $i \in K$. Player I wins if $x \in E$ and Player II — if $x \notin E$. Let $V_{II}(K)$ be the set of all $E \subseteq 2^\omega$ such that Player II has a winning strategy in $\Gamma(E, K)$. Now, consider a system $\{K_s : s \in 2^{<\omega}\}$ (where $2^{<\omega}$ denotes the set of all finite sequences with terms from $\{0, 1\}$) fulfilling the conditions

$$K_{s0} \cup K_{s1} \subseteq K_s \quad \text{and} \quad K_{s0} \cap K_{s1} = \emptyset$$

for all $s \in 2^{<\omega}$ where si ($i \in \{0, 1\}$) extends s by the (last) term i. The family

$$\mathcal{M} = \cap\{V_{II}(K_s) : s \in 2^{<\omega}\}$$

is an ideal defined by Mycielski in [**My**]. It is interesting that there exists a set E in $\mathcal{M}$ such that $2^\omega \setminus E$ is of the first category and of measure zero (in 2^ω we consider the usual product measure which is isomorphic to the Lebesgue measure on $[0, 1]$). The ideal $\mathcal{M}$ has a base consisting of G_δ sets. The above facts are observed in [**My**]. Thus we have $\mathrm{cof}\,(\mathcal{M}) \le c$. It was proved in [**Ba1**] that $\mathcal{M}$ has property (P). Rosłanoski showed that $\mathrm{cov}\,(\mathcal{M}) = \omega_1$ (see [**R,** Th. 2.3(a)]). Hence, if we assume CH, Criterion 1 yields $\mathcal{I}_0 \setminus \mathcal{M} \neq \emptyset$. (Can it be proved within ZFC?)

Now we give an example of an ideal for which Criterion 1 works in ZFC.

1.3. Example. Let $\mathcal{F}$ be a disjoint family of perfect sets with the union equal to $X = 2^\omega$, we shall define sets Q_α, $\alpha < c$. Let $\mathcal{R}$ be the family of all sets $Q \subseteq X$ such that $Q \cap P$ is countable for any $P \in \mathcal{F}$. If $\mathcal{R} = \emptyset$, put $Q_\alpha = \emptyset$ for all $\alpha < c$. If $\mathcal{R} \neq \emptyset$; pick any $Q_0 \in \mathcal{R}$. Assume that $\alpha < c$ and Q_γ for $\gamma < \alpha$ are defined. If there is a $Q \in \mathcal{R}$ such that $Q \cap Q_\gamma$ is countable for all $\gamma < \alpha$, put

$Q_\alpha = Q$, and let $Q_\alpha = \emptyset$ in the opposite case. Next, put $\mathcal{F}^+ = \mathcal{F} \cup \{Q_\alpha : \alpha < c\}$ and $\mathcal{I} = \{E \subseteq X : E \subseteq \cup\tilde{\mathcal{F}}$ for some finite $\tilde{\mathcal{F}} \subseteq \mathcal{F}^+\}$. Then $\operatorname{cof}(\mathcal{I}) \leq c$ since $|\{\tilde{\mathcal{F}} \subseteq \mathcal{F}^+ : \tilde{\mathcal{F}}$ is finite$\}| = c$. Now, observe that $\operatorname{cov}(\mathcal{I}) = c$. Indeed, if $\operatorname{cov}(\mathcal{I}) = \kappa < c$, there is an $\mathcal{F}_0 \subseteq \mathcal{F}^+$ such that $|\mathcal{F}_0| = \kappa$ and $\cup\mathcal{F}_0 = X$. Let $\mathcal{F}_0 = \{P_\alpha : \alpha < \kappa\}$. Consider a fixed $P \in \mathcal{F}\setminus\mathcal{F}_0$. $P \cap P_\alpha$ is countable for all $\alpha < \kappa$, therefore $c = |P| = |P \cap \cup F_0| = |\cup_{\alpha<\kappa} P \cap P_\alpha| \leq \kappa \cdot \omega = \kappa < c$, a contradiction. The ideal $\mathcal{I}$ has property (P) since, by the construction, each perfect set P either belongs to $\mathcal{F}^+$, thus is in $\mathcal{I}$, or $P \cap Q$ is uncountable for some $Q \in \mathcal{F}^+$, thus a perfect part of $P \cap Q$ belongs to $\mathcal{I}$.

2. The Second Criterion

For a family $\{D_\alpha : \alpha < c\} \subseteq \mathcal{P}(X)$, we denote

$$D_0^* = D_0 \quad \text{and} \quad D_\alpha^* = D_\alpha \setminus \cup_{\alpha<\gamma} D_\gamma \text{ if } 0 < \alpha < c\,.$$

The following proposition generalizes Theorem 2.2 from [**W1**].

2.1. Proposition. *Let $\mathcal{D} = \{D_\alpha : \alpha < c\}$ be a family of analytic subsets of X such that $|D_\alpha^*| = c$ for all $\alpha < c$. Then there exists a selector E of $\{D_\alpha^* : \alpha < c\}$ being an (s^0) set.*

Proof. Let $\mathcal{P}$ be the family of all perfect subsets of X. If there exists $P \in \mathcal{P}$ meeting each member of D_α at $< c$ points (consequently, in a countable set of points), then let $\{Q_\alpha : \alpha < c\}$ consists of all such sets P. In the opposite case, let $Q_\alpha = \emptyset$ for all $\alpha < c$. Pick $x_0 \in D_0$ and choose inductively

$$x_\alpha \in D_\alpha^* \setminus \cup_{\gamma<\alpha}(Q_\gamma \cup \{x_\gamma\}) \quad \text{for } 0 < \alpha < c\,.$$

This can be done since $|D_\alpha^*| = c$ and $|D_\alpha^* \cap (\cup_{\gamma<\alpha}(Q_\gamma \cup \{x_\gamma\})| < c$. Define $E = \{x_\alpha : \alpha < c\}$. Certainly, $E \cap D_\alpha^* = \{x_\alpha\}$ for each $\alpha < c$. Now, consider any perfect P. If $P = Q_\alpha$ for some $\alpha < c$, then $E \cap P \subseteq \{x_\beta : \beta \leq \alpha\}$. So, by Lemma 0.1, there is a perfect subset of P which misses E. If $P \neq Q_\alpha$ for all $\alpha < c$, then $|P \cap D_\alpha| = c$ for some $\alpha < c$. For this α, we have

$$P \cap D_\alpha \cap E \subseteq \{x_\beta : \beta \leq \alpha\}\,.$$

So, by Lemma 0.1, there is a perfect subset of $P \cap D_\alpha$ (consequently, of P) disjoint from E. Hence E is an (s^0) set. □

Let $\mathcal{D} \subseteq \mathcal{P}(X)$, $|\mathcal{D}| = c$ and $\cup\mathcal{D} = X$. We say that an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ is $(< c)$-generated by $\mathcal{D}$ if

$$\mathcal{I} = \{E \subseteq X : E \subseteq \cup\tilde{\mathcal{D}} \text{ for some } \tilde{\mathcal{D}} \subseteq \mathcal{D}, |\tilde{\mathcal{D}}| < c\}\,.$$

2.2. Corollary (Criterion 2). *If $\mathcal{D} = \{D_\alpha : \alpha < c\}$ is a family of analytic subsets of X such that $\cup\mathcal{D} = X$ and $|D^*_\alpha| = c$ for all $\alpha < c$, then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}$ is the ideal $(< c)$-generated by $\mathcal{D}$.*

Proof. Consider the set E from Proposition 2.1. Then $E \in \mathcal{I}_0$ and, since E is a selector of $\{D^*_\alpha : \alpha < c\}$, we get $E \notin \mathcal{I}$. □

We say that a family $\mathcal{F}$ of perfect subsets of X is almost disjoint if $P \cap Q$ is countable for any distinct $P, Q \in \mathcal{F}$. By Zorn's lemma, each almost disjoint family of perfect sets can be extended to a maximal one. From Criterion 2 we get

2.3. Corollary. *If $\mathcal{D}$ is an almost disjoint family of perfect subsets of X, such that $|\mathcal{D}| = c$ and $\cup\mathcal{D} = X$, then $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$ where $\mathcal{I}$ is the ideal $(< c)$-generated by $\mathcal{D}$.*

2.4. Examples. (a) Let $I = [0, 1]$ and $X = I^2$. Put

$$\mathcal{D} = \{I \times \{x\} : x \in I\} \cup \{\{x\} \times I : x \in I\}\,.$$

Then $\mathcal{D}$ is an almost disjoint family of perfect sets fulfilling the assumptions of 2.3. Note that $\mathcal{D}$ is not maximal since, for instance, the diagonal meets each set from $\mathcal{D}$ at exactly one point. By that reason, property (P) fails to hold for the ideal $\mathcal{I}$ $(< c)$-generate by $\mathcal{D}$ since the diagonal has no perfect subset in $\mathcal{I}$. So, Criterion 1 cannot be applied to $\mathcal{I}$.

(b) Let P be a perfect subset on $\mathbb{R}$ such that $|P \cap (P + x)| \le 1$ for all $x \neq 0$ (here $P + x$ denotes the set of all sums $t + x$ for $t \in P$); see [**Ru-S**]. Then, for any perfect $Q \subseteq P$, the collection $\mathcal{D} = \{Q + x : x \in \mathbb{R}\}$ is an almost disjoint family of perfect sets fulfilling all the assumptions of 2.3. If there exists a perfect $S \subseteq P \setminus Q$, it is clear that $\mathcal{D}$ is not maximal since $\mathcal{D} \cup \{S\}$ extends $\mathcal{D}$. Hence again, the respective ideal $\mathcal{I}$ has not property (P). Note that $\mathcal{I}$ is translation invariant.

(c) Observe that $\mathcal{F}^+$ from Example 1.3 can form a maximal almost disjoint family of perfect sets. By 2.3, there is an (s^0) set outside the ideal $(< c)$-generated by $\mathcal{F}^+$. That ideal contains $\mathcal{I}$ considered in 1.3. Thus, now we get more that $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$.

(d) Let X be the set of all infinite subsets of ω. Then X can be embedded into the Cantor set 2^ω via the characteristic functions. Thus X inherits the product topology from 2^ω and forms a dense-in-itself space which is Polish since it is embedded into 2^ω as a G_δ set (apply the Alexandrov theorem, see [**Kr,** §33.VI]). Let $A \subseteq X$ be a family of c sets which meet pairwise on finite sets (see [**Kn,** Th. 1.2(b), p. 48]). Let $\mathcal{A} = \{A_\alpha : \alpha < c\}$ and $D_\alpha = \{K \in X : K \cap A_\alpha \in X\}$, $\alpha < c$. It is easy to verify that $\mathcal{D} = \{D_\alpha : \alpha < c\}$ consists of perfect sets and $\cup\mathcal{D} = X$. This is not an almost disjoint family since $|D_\alpha \cap D_\beta| = c$ for any distinct $\alpha, \beta < c$. Indeed, there exist c distinct subsets of ω meeting either of the sets A_α and A_β in infinite sets. On the other hand, Criterion 2 can be applied to $\mathcal{D}$ since $|D^*_\alpha| = c$ for $\alpha < c$. This follows from the fact that A_α has c infinite subsets and each of them is in D^*_α.

Finally, note that, for any ideal $\mathcal{I}$ fulfilling the conditions of Criterion 2, it is consistent with ZFC that $\mathrm{cf}\,(\mathcal{I}) > c$. Indeed, we have $\mathrm{cf}\,(\mathcal{I}) \geq 2^{\omega_1}$ hence $\mathrm{cf}\,(\mathcal{I}) > c$ holds in the model in which $\omega_1 < c$ and $2^{\omega_1} > c$ are true (see [**Kn,** Th. 6.18(c), p. 216]). So, in this case, Criterion 1 is not useful.

3. Further Remarks

In Sections 1 and 2 we have concentrated on the problem "When $\mathcal{I}_0 \setminus \mathcal{I} \neq \emptyset$?", while the results for Brown and Walsh which we try to generalize deal mainly with the question "When $\mathcal{I}_0 \setminus S_{\mathcal{I}} \neq \emptyset$?" where $S_{\mathcal{I}}$ is a respective σ-field associated with $\mathcal{I}$. Now, we shall show that, in some cases, these two problems are equivalent.

For a family $\mathcal{F} \subseteq \mathcal{P}(X)$ and an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$, by $\mathcal{F}(\mathcal{I})$ we denote the collection of all sets $E \subseteq X$ expressible as the symmetric differences $B \triangle C$ where $B \in \mathcal{F}$ and $A \in \mathcal{I}$. In particular, one can consider as $\mathcal{F}$ the σ-field $\mathcal{B}$ of all Borel sets in X; then $\mathcal{B}(\mathcal{I})$ is the smallest σ-field containing $\mathcal{B} \cup \mathcal{I}$. We shall also consider projective pointclasses Σ^1_n and Π^1_n for $n \geq 1$ (see [**Mo**], for the definitions); here we restrict them to the space X. We say that a pointclass Λ fulfils Perfect Set Theorem (abbr. PST) if each uncountable set from Λ contains a perfect set. It is known that Σ^1_1 fulfils PST and, for $n \geq 2$, the statement "Σ^1_n fulfils PST", is not provable in ZFC; however, it can be treated as a strong axiom of set theory (cf. [**Mo**]).

For $\mathcal{F} \subseteq \mathcal{P}(X)$, we denote $\neg\mathcal{F} = \{X \setminus A : A \in \mathcal{F}\}$.

3.1. Proposition. *Assume that $\mathcal{F} \subseteq \mathcal{P}(X)$ is closed under finite intersections and $\mathcal{I} \subseteq \mathcal{P}(X)$ is an ideal with a base $\tilde{\mathcal{I}} \subseteq \neg\mathcal{F}$ such that each set from $\mathcal{F} \setminus \mathcal{I}$ contains a perfect set. Let $E \subseteq X$ be totally imperfect. Then $E \notin \mathcal{I}$ and $E \notin \mathcal{F}(\mathcal{I})$ are equivalent.*

Proof. Obviously, $E \notin \mathcal{F}(\mathcal{I})$ implies $E \notin \mathcal{I}$. Now, assume that we have a totally imperfect $E \in \mathcal{I}$. Suppose that $E \in \mathcal{F}(\mathcal{I})$. Then $E = B \triangle A$ where $B \in \mathcal{F}$ and $A \in \mathcal{I}$. Of course, $B \notin \mathcal{I}$. Choose $\tilde{A} \in \tilde{\mathcal{I}}$ containing A. Then, for $\tilde{B} = B \setminus \tilde{A}$, we get $E = \tilde{B} \cup D$ where $D = (B \cap (\tilde{A} \setminus A)) \cup (A \setminus B) \in \mathcal{I}$. Observe that $\tilde{B}$ is in $\mathcal{F} \setminus \mathcal{I}$ and thus, by the assumption, it contains a perfect set. Hence E has a perfect subset, a contradiction. $\square$

If an ideal $\mathcal{I} \subseteq \mathcal{P}(X)$ has a base $\tilde{\mathcal{I}}$ contained in a pointclass Λ, then $\tilde{\mathcal{I}}$ is called a Λ-base.

3.2. Corollary. *Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal having a Π^1_1-base and containing all countable subsets of X. For any totally imperfect set E, the conditions $E \notin \mathcal{I}$ and $E \notin \Sigma^1_1(\mathcal{I})$ are equivalent.*

The same result holds when Π_1^1 and Σ_1^1 are replaced by $\mathcal{B}$.

3.3. Corollary. *For $n \geq 2$, assume that Σ_n^1 fulfils PST. Let $\mathcal{I} \subseteq \mathcal{P}(X)$ be an ideal having a Π_n^1-base and containing all countable subsets of X. For any totally imperfect set, the conditions $E \notin \mathcal{I}$ and $E \notin \Sigma_n^1(\mathcal{I})$ are equivalent.*

Applying the results of this section together with Criterion 1 or 2 to the respective ideals $\mathcal{I}$ and pointclasses Λ, we get conditions guaranteeing $\mathcal{I}_0 \setminus \Lambda(\mathcal{I}) \neq \emptyset$.

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