

ON HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM  
EQUIAFFINE GENERALLY RECURRENT SPACES ONTO  
KÄHLERIAN SPACES

RAAD J.K. AL LAMI, MARIE ŠKODOVÁ, JOSEF MIKEŠ

ABSTRACT. In this paper we consider holomorphically projective mappings from the special generally recurrent equiaffine spaces  $A_n$  onto (pseudo-) Kählerian spaces  $\bar{K}_n$ . We proved that these spaces  $A_n$  do not admit nontrivial holomorphically projective mappings onto  $\bar{K}_n$ .

These results are a generalization of results by T. Sakaguchi, J. Mikeš and V. V. Domashev, which were done for holomorphically projective mappings of symmetric, recurrent and semisymmetric Kählerian spaces.

1. INTRODUCTION

In this paper we present some new results obtained for holomorphically projective mappings from equiaffine special spaces  $A_n$  onto Kählerian spaces  $\bar{K}_n$ .

These  $A_n$  are generally recurrent, including  $m$ -recurrent ( $K_n^m$ ) in the sense of V. R. Kaygorodov [5, 6]. It is known that if the spaces  $K_n^m$  are (pseudo-) Riemannian spaces  $V_n$  (briefly – *Riemannian*) then they are semisymmetric.

An  $n$ -dimensional manifold  $A_n$  with affine connection  $\nabla$  is an *equiaffine space* if in  $A_n$  the Ricci tensor  $Ric$  is symmetric. These spaces are characterized by a coordinate system  $x$  such that  $\Gamma_{\alpha i}^{\alpha}(x) = \partial f(x)/\partial x^i$ , where  $f(x)$  is a function on  $A_n$ , and  $\Gamma_{ij}^h(x)$  are components of a connection  $\nabla$  [3, 13, 20, 23, 27].

A Riemannian space  $\bar{K}_n$  is called a *Kählerian space* if it is endowed, besides a metric tensor  $\bar{g}$ , with an affinor structure  $F$  satisfying the following relations [4, 14, 23, 27]

$$F^2 = -\text{Id}, \quad \bar{g}(X, FX) = 0, \quad \bar{\nabla}F = 0.$$

Here  $X$  are all tangent vectors of  $T\bar{K}_n$  and  $\bar{\nabla}$  is a connection of  $\bar{K}_n$ . The structure  $F$  is a complex structure.

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## 2. HOLOMORPHICALLY PROJECTIVE MAPPINGS

An  $F$ -planar curve of a space  $A_n$  with an affinor structure  $F$  is a curve  $x = x(t)$  whose tangent vector  $\lambda(t) = dx(t)/dt$ , being translated, remains in the area element formed by the tangent vector  $\lambda$  and its conjugate vector  $F\lambda$ , i.e., the conditions

$$\nabla_\lambda \lambda = \rho_1(t) \lambda + \rho_2(t) F\lambda,$$

where  $\rho_1, \rho_2$  are functions of the argument  $t$ , are fulfilled [14, 18].

In Kählerian and Hermitian spaces with a structure  $F$  these curves are called *analytically planar* [14, 19, 23, 27].

A diffeomorphism of  $A_n$  onto  $\bar{A}_n$  is called an  $F$ -planar mapping if it maps all  $F$ -planar curve of  $A_n$  into  $\bar{F}$ -planar curve of  $\bar{A}_n$  [14, 18].

If the structures  $F$  and  $\bar{F}$  are (almost) complex structures then  $F$ -planar mappings are evidently *holomorphically projective mappings*. These mappings for Kählerian and Hermitian spaces have been studied by many authors, see [2, 8, 11, 14, 15, 16, 18, 19, 21, 23, 24, 27].

Consider a concrete mapping  $f: A_n \rightarrow \bar{K}_n$ , both spaces being referred to a common coordinate system  $x$  with respect to this mapping. This is a coordinate system where two corresponding points  $M \in A_n$  and  $f(M) \in \bar{K}_n$  have equal coordinates  $x = (x^1, x^2, \dots, x^n)$ ; the corresponding geometric objects in  $\bar{K}_n$  will be marked with a bar. For example,  $\Gamma_{ij}^h$  and  $\bar{\Gamma}_{ij}^h$  are components of the affine connection  $\nabla$  on  $A_n$  and  $\bar{\nabla}$  on  $\bar{K}_n$ , respectively.

An equiaffine space  $A_n$  admits a *holomorphically projective mapping*  $f$  onto a Kählerian space  $\bar{K}_n$  if and only if

$$(1) \quad \bar{\nabla}_X Y = \nabla_X Y + \psi(X)Y + \psi(Y)X - \psi(FX)FY - \psi(FY)FX,$$

where  $\forall X, Y \in TA_n$ ,  $\psi$  is a closed linear form on  $A_n$ , i.e.  $\psi(X) = X\psi(x)$ ,  $\psi(x)$  is a function on  $A_n$ .

If the linear form  $\psi \neq 0$ , then a holomorphically projective mapping is called *nontrivial*; otherwise it is said to be *trivial* or *affine*. A complex structure  $F$  on a space  $A_n$  is necessary also covariantly constant, i.e.  $\nabla F = 0$ .

Further we will use local coordinates  $x$  on a chart  $(x, U) \subset A_n$ .

The formula (1) in this chart has the following expression:

$$(2) \quad \bar{\Gamma}_{ij}^h(x) = \Gamma_{ij}^h(x) + \delta_{(i}^h \psi_{j)} - F_{(i}^h F_{j)}^\alpha \psi_\alpha,$$

where  $\Gamma_{ij}^h$  and  $\bar{\Gamma}_{ij}^h$  are components of  $\nabla$  and  $\bar{\nabla}$ , respectively,  $\psi_i = \partial_i \psi(x)$  are components of linear form  $\psi$ ,  $F_i^h(x)$  are components of  $F$ ,  $\delta_i^h$  is the Kronecker symbol, and  $(ij)$  denotes the symmetrization without division.

The following theorem holds [15]:

**Theorem 1.** *Let in an equiaffine space  $A_n$  exist the solution of the following system of linear differential equations with respect to the unknown functions  $a^{ij}(x)$  and  $\lambda^i(x)$ :*

$$(3) \quad a^{ij},{}_k = \lambda^i \delta_k^j + \lambda^j \delta_k^i + \lambda^\alpha F_\alpha^i F_k^j + \lambda^\alpha F_\alpha^j F_k^i,$$

where “,” denotes the covariant derivative with respect to the connection  $\nabla$  of the space  $A_n$ , the matrix  $\|a^{ij}\|$  should further satisfy  $\det \|a^{ij}\| \neq 0$  and the algebraic conditions  $a^{ij} = a^{ji}$  and  $a^{ij} = a^{\alpha\beta} F_\alpha^i F_\beta^j$ .

Then  $A_n$  admits a holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$ . The metric tensor  $\bar{g}_{ij}$  of  $\bar{K}_n$  and solutions of (3) are connected by the relations

$$(4) \quad \text{a) } a^{ij} = e^{-2\psi(x)} \bar{g}^{ij}, \quad \text{b) } \lambda^i = -a^{i\alpha} \partial_\alpha \psi(x),$$

where  $\bar{g}^{ij}$  are components of inverse matrix of  $\|\bar{g}_{ij}\|$ .

This theorem is a generalization of results in [2, 14, 23].

The question of existence of a solution of (3) leads to the study of integrability conditions and their differential prolongations. The general solution of (3) does not depend on more than  $N_o = 1/4(n + 1)^2$  parameters [15].

Let in an equiaffine space  $A_n$  the condition for Riemannian (curvature) tensor

$$(5) \quad R_{ijk}^h = \delta_i^h \overset{1}{v}_{jk} + \delta_j^h \overset{2}{v}_{ik} - \delta_k^h \overset{2}{v}_{ij} + F_i^h \overset{3}{v}_{jk} + F_j^h \overset{4}{v}_{ik} - F_k^h \overset{4}{v}_{ij}$$

hold, where  $\overset{\sigma}{v}$  are tensors.

**Lemma 1.** *If an equiaffine space  $A_n$  with the condition (5) admits a holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$ , then  $\bar{K}_n$  has constant holomorphic curvature.*

This space  $A_n$  is called a *holomorphically projective flat space*.

**Proof.** In [7, 12] an  $F$ -traceless decomposition of Riemannian tensor is studied. Formula (5) is this decomposition, in which  $F$ -traceless tensor vanishes.

The Riemannian tensor  $\bar{R}_{ijk}^h$  of Kählerian space  $\bar{K}_n$ , onto which  $A_n$  is holomorphically projective mapped, satisfies an analogical form as (5). From [12], under this condition and the uniqueness of this decomposition one can show that a tensor of holomorphic projective curvature of  $\bar{K}_n$  vanishes. This is a criterion for  $\bar{K}_n$  to have constant holomorphic curvature, see [14, 23, 27].  $\square$

### 3. HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM SEMISYMMETRIC EQUIAFFINE SPACES

Hereafter we shall assume that in the equiaffine space  $A_n$  the Ricci tensor will be preserved under the action of the structure  $F$ , i.e.

$$Ric(FX, FY) = Ric(X, Y).$$

We remind that the condition  $\nabla F = 0$  implies certain properties for the Riemannian tensor, for example:

$$F^h_\alpha R_{ijk}^\alpha = F_i^\alpha R_{\alpha jk}^h, \quad F_\alpha^h F_i^\beta R_{\beta jk}^\alpha = -R_{ijk}^h.$$

These formulas naturally hold on Kählerian spaces.

The affine-connected spaces  $A_n$  are called *semisymmetric* if the condition  $R \cdot R = 0$  holds, which, in coordinate notation, has the form  $R_{ijk,[lm]}^h = 0$ . According to the Ricci identity, this condition is written as follows

$$R_{\alpha jk}^h R_{ilm}^\alpha + R_{iak}^h R_{jlm}^\alpha + R_{ij\alpha}^h R_{klm}^\alpha - R_{ijk}^\alpha R_{alm}^h = 0.$$

Many investigations are devoted to the study of these spaces, see [1, 5, 6, 13, 14, 21, 23, 25].

**Theorem 2** (M. Škodová, J. Mikeš, O. Pokorná [24]). *Let an equiaffine semisymmetric space  $A_n$ , where the Ricci tensor under the action of the structure  $F$  will be preserved, admit nontrivial holomorphically projective mappings onto a Kählerian space  $\bar{K}_n$ . If  $A_n$  is not a holomorphically projective flat space then the vector field  $\lambda^h$  from the equation (3) is convergent, i.e.  $\lambda^h_{,i} = \text{const} \cdot \delta^h_i$  is satisfied.*

Analogical results were proved by J. Mikeš for the geodesic mappings of semisymmetric Riemannian spaces and space with affine connection, see [10, 13, 23], and for holomorphically projective mappings of semisymmetric Kählerian spaces, see [14].

In the following we will study the holomorphically projective mapping  $A_n$  onto a Kählerian space  $\bar{K}_n$ , on the assumption that  $\lambda^h$  is a *conccircular vector field* (in sense of K. Yano [26], see [13, 17, 23]), i.e. it holds

$$(6) \quad \lambda^h_{,i} = \varrho \delta^h_i,$$

where  $\varrho$  is a function.

The conditions of integrability (6) have the form:  $\lambda^\alpha R^h_{\alpha j k} = \varrho_{,j} \delta^h_k - \varrho_{,k} \delta^h_j$ . Because if in a space  $A_n$   $R^h_{i\alpha\beta} F_j^\alpha F_k^\beta = R^h_{i j k}$  holds then  $\varrho = \text{const}$ , i.e.  $\lambda^h$  is *convergent*.

If we covariantly differentiate (4b) and (6) then we obtain the following formula

$$(7) \quad \psi_{ij} = \Delta \bar{g}_{ij},$$

where  $\Delta$  is a function and  $\psi_{ij} = \psi_{i,j} - \psi_i \psi_j + \psi_\alpha \psi_\beta F_i^\alpha F_j^\beta$ . We make sure by the analysis of the identity  $\psi_\alpha R^{\alpha}_{ij k} = \psi_{i,j k} - \psi_{i,k j}$ , that  $\Delta \equiv \text{const}$ .

The formulas (6) and (7) are equivalent.

From equations (2) and (7) for the Riemannian and Ricci tensors of  $A_n$  and  $\bar{K}_n$  this follows

$$\begin{aligned} \bar{R}^h_{ijk} &= R^h_{ijk} - \Delta (\delta^h_k \bar{g}_{ij} - \delta^h_j \bar{g}_{ik} + F_k^h F_i^\alpha \bar{g}_{\alpha j} - F_j^h F_i^\alpha \bar{g}_{\alpha k} + 2F_i^h F_j^\alpha \bar{g}_{\alpha k}), \\ \bar{R}_{ij} &= R_{ij} + (n + 2)\Delta \bar{g}_{ij}. \end{aligned}$$

We can make sure that the integrability conditions of equations (3) and (6) and their prolongations have the following forms:

$$(8_0) \lambda^\alpha R^h_{\alpha j k} = 0, \dots, (8_m) \lambda^\alpha R^h_{\alpha j k, l_1 \dots l_m} = \varrho \overset{m}{T}{}^h_{j k l_1 \dots l_m},$$

$$(9_0) a^{\alpha(i} R^j)_{\alpha k l} = 0, \dots, (9_m) a^{\alpha(i} R^j)_{\alpha k l, l_1 \dots l_m} = \lambda^{(i} \overset{m}{T}{}^j)_{k l l_1 \dots l_m} + \lambda^\alpha F_\alpha^{(i} F^j) \overset{m}{T}{}^\beta_{k l l_1 \dots l_m},$$

where the tensor  $\overset{m}{T}$  is determined by the formulas

$$\overset{m}{T}{}^h_{j k l_1 \dots l_m} \stackrel{\text{def}}{=} - \sum_{s=1}^m R^h_{l_s j k, l_1 \dots l_{s-1} l_{s+1} \dots l_m}.$$

4. HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM GENERALLY RECURRENT EQUIAFFINE SPACES ONTO KÄHLERIAN SPACES

As it is known, *symmetric* and *recurrent* spaces  $A_n$  are characterized by differential conditions on the Riemannian tensor  $R^h_{ijk,l} = 0$  and  $R^h_{ijk,l} = \varphi_l R^h_{ijk}$ , respectively, where  $\varphi_l \neq 0$  is a covector.

Holomorphically projective mappings of symmetric and recurrent Kählerian spaces were studied by T. Sakaguchi [21], J. Mikeš and V. V. Domashev [2, 14, 23]. These results are generalized in the following theorem [24]:

**Theorem 3.** *Let an equiaffine symmetric (or semisymmetric recurrent) space  $A_n$ , where the Ricci tensor under the action of the structure  $F$  will be preserved, admit a nontrivial holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$ . Then  $A_n$  is holomorphically projectively flat and the space  $\bar{K}_n$  has constant holomorphic curvature.*

This theorem is possible to generalize for such  $A_n$ , which have more general recurrences of the Riemannian tensor.

**Theorem 4.** *Let  $A_n$  be an equiaffine space, where the Ricci tensor under the action of the structure  $F$  will be preserved, and one of these two conditions holds in  $A_n$ :*

$$(10) \quad R^h_{ijk,l} = R^h_{i\alpha\beta} S^{\alpha\beta}_{jkl};$$

$$(11) \quad Q^{hi}_{(pr)jk,l} = Q^{\gamma\delta}_{(pr)\alpha\beta} S^{hi\alpha\beta}_{\gamma\delta jkl},$$

where  $S$  are certain tensors and

$$Q^{hi}_{prjk} \stackrel{\text{def}}{=} \delta^h_p R^i_{rjk} + \delta^i_p R^h_{rjk} + F^h_p F^i_\alpha R^\alpha_{rjk} + F^i_p F^h_\alpha R^\alpha_{rjk}.$$

If  $A_n$  admits a nontrivial holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$  and the condition (6) holds then  $A_n$  is flat, and the space  $\bar{K}_n$  has constant holomorphic curvature.

**Proof.** Let  $A_n$  admit a nontrivial holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$  and assume that condition (6) holds. Hence the conditions (9) hold.

And it is easy to see that the conditions (11) follow from (10). We contract (11) with  $a^{pr}$ . According to formulas (9<sub>0</sub>) and (9<sub>1</sub>) we obtain

$$\lambda^{(h} R^i_{ljk} + \lambda^\alpha F^\alpha_{(h} F^i_{\beta)} R^\beta_{ljk} = 0.$$

From these formulas on  $\lambda^h \neq 0$  it follows that  $R^h_{ijk} = 0$ , i.e.  $A_n$  is flat, and according to Lemma 1 a space  $\bar{K}_n$  has constant holomorphic curvature.  $\square$

5. HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM  $m$ -RECURRENT EQUIAFFINE SPACES ONTO KÄHLERIAN SPACES

We mention the next definitions (V. R. Kaygorodov [5, 6]):

The space  $A_n$  is called an  $m$ -recurrent space ( $K_n^m$ ) if

$$(12) \quad R_{ijk,l_1 \dots l_m}^h = \Omega_{l_1 \dots l_m} R_{ijk}^h,$$

where  $\Omega$  is a nonvanishing tensor.

All (pseudo-) Riemannian  $m$ -recurrent spaces  $K_n^m$  are semisymmetric [5, 6].

In the sequel we shall need the following Lemmas.

**Lemma 2.** *Let*

$$(13) \quad A_{l_1} \omega_{l_2 l_3 \dots l_m} + A_{l_2} \omega_{l_1 l_3 \dots l_m} + \dots + A_{l_m} \omega_{l_1 l_2 \dots l_{m-1}} = 0,$$

hold for a covector  $A$  and a tensor  $\omega$ . If the tensor  $\omega$  is nonvanishing then  $A = 0$ .

**Proof.** Let formulas (13) hold and let the tensor  $\omega$  be not vanishing.

We contract the tensor  $\omega_{l_2 l_3 \dots l_m}$  with a vector  $b^{l_m}$ , for which

$$\omega_{l_2 l_3 \dots l_{m-1}} \stackrel{\text{def}}{=} b^\alpha \omega_{l_2 l_3 \dots l_{m-1} \alpha} \neq 0,$$

holds. From (13) we obtain

$$(14) \quad \begin{aligned} & A_{l_1} \omega_{l_2 l_3 \dots l_{m-1}} + A_{l_2} \omega_{l_1 l_3 \dots l_{m-1}} + \dots + A_{l_{m-1}} \omega_{l_1 l_2 \dots l_{m-2}} \\ & + b^\alpha A_\alpha \omega_{l_1 l_2 \dots l_{m-1}} = 0. \end{aligned}$$

Hence it follows that

$$b^\alpha A_\alpha = 0.$$

If  $b^\alpha A_\alpha \neq 0$  held we would obtain after the contraction (14) with  $b^{l_{m-1}}$

$$\begin{aligned} & A_{l_1} \omega_{l_2 l_3 \dots l_{m-2}} + A_{l_2} \omega_{l_1 l_3 \dots l_{m-2}} + \dots + A_{l_{m-2}} \omega_{l_1 l_2 \dots l_{m-3}} \\ & + 2b^\alpha A_\alpha \omega_{l_1 l_2 \dots l_{m-2}} = 0, \end{aligned}$$

where

$$\omega_{l_2 l_3 \dots l_{m-2}} \stackrel{\text{def}}{=} b^{l_{m-1}} \omega_{l_2 l_3 \dots l_{m-1}}.$$

Because  $\omega_{l_1 l_2 \dots l_{m-2}} \neq 0$  and  $b^\alpha A_\alpha \neq 0$  then  $\omega_{l_1 l_2 \dots l_{m-3}} \neq 0$ .

We continue step-by-step, at last we obtain  $A_{l_1} \omega_{l_2 \dots l_m} + (m-1) \omega_{l_1} b^\alpha A_\alpha = 0$ , where

$\omega_{l_1} \neq 0$ . We contract the last term with  $b^{l_1}$  and we can see that  $m \omega_{l_1} b^\alpha A_\alpha = 0$ , i.e.  $b^\alpha A_\alpha = 0$  (because  $\omega_{l_1} \neq 0$ ), i.e. it is a contradiction. Hence  $b^\alpha A_\alpha = 0$ .

Thus the formula (14) takes the form

$$A_{l_1} \omega_{l_2 l_3 \dots l_{m-1}} + A_{l_2} \omega_{l_1 l_3 \dots l_{m-1}} + \dots + A_{l_{m-1}} \omega_{l_1 l_2 \dots l_{m-2}} = 0,$$

where  $\omega_{l_2} \neq 0$ . The obtained formulas are of the form (14), with a difference, that the tensor  $\omega_{l_2}$  has a valence lower by one than the tensor  $\omega$ . We continue step by step until it is

$$A_{l_1} \tilde{\omega}_{l_2} + A_{l_2} \tilde{\omega}_{l_1} = 0,$$

where  $\tilde{\omega}_l$  is a nonvanishing tensor. Hence it follows that  $A_i = 0$ . □

**Lemma 3.** *Let*

$$(15) \quad R_{l_1 j k}^h \omega_{l_2 l_3 \dots l_m} + R_{l_2 j k}^h \omega_{l_1 l_3 \dots l_m} + \dots + R_{l_m j k}^h \omega_{l_1 l_2 \dots l_{m-1}} = 0,$$

*hold for the Riemannian tensor of  $A_n$ . If the tensor  $\omega$  is nonvanishing then  $A_n$  is flat.*

**Proof.** Let  $A_n$  be non-flat then  $R_{i j k}^h \neq 0$ . Therefore a tensor  $\varepsilon_h^{j k}$  exists such that  $A_i = \varepsilon_h^{j k} R_{i j k}^h \neq 0$ . We contract (15) with  $\varepsilon_h^{j k}$  and obtain the formula (13) and Lemma 3 holds according to Lemma 2.  $\square$

The following holds

**Theorem 5.** *Let an equiaffine  $m$ -recurrent space  $K_n^m$ , where the Ricci tensor under the action of the structure  $F$  will be preserved, admit a nontrivial holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$  and the condition (6) holds.*

*Then  $K_n^m$  is flat and the space  $\bar{K}_n$  has constant holomorphic curvature.*

**Proof.** Let the space  $K_n^m$  admit a nontrivial holomorphically projective mapping onto a Kählerian space  $\bar{K}_n$  and assume that condition (6) holds. Contracting (12) with  $\lambda^i$  and using (8<sub>0</sub>) and (8<sub>m</sub>) we obtain

$$\varrho T_{kl l_1 \dots l_m}^m = 0.$$

We assume that  $\varrho \neq 0$ . Then  $T_{kl l_1 \dots l_m}^m = 0$ . We covariantly differentiate along  $x^l$  apply to (12) and obtain these formulas

$$(16) \quad R_{l_1 j k}^h \Omega_{l_2 l_3 \dots l_m l} + R_{l_2 j k}^h \Omega_{l_1 l_3 \dots l_m l} + \dots + R_{l_m j k}^h \Omega_{l_1 l_2 \dots l_{m-1} l} = 0.$$

Because the tensor  $\Omega \neq 0$  the vector  $\varepsilon^l$  exists such that

$$(17) \quad \omega_{l_2 l_3 \dots l_m} = \varepsilon^l \Omega_{l_2 l_3 \dots l_m l} \neq 0.$$

Contracting (16) with  $\varepsilon^l$  we obtain the formula (15). Because  $A_n$  is not flat ( $R_{i j k}^h \neq 0$ ), from Lemma 3 it follows that  $\varrho = 0$ , thus the vector  $\lambda^i$  is covariantly constant, i.e.  $\lambda_{,i}^h = 0$ .

Let us contract (12) with  $a^{ir}$  and after it let us alternate the indices  $r$  a  $h$ . After application of (9<sub>0</sub>) and (9<sub>m</sub>) we can obtain

$$(18) \quad \lambda^{(i} T_{kl l_1 \dots l_m}^{m j)} + \lambda^\alpha F_\alpha^{(i} F_\beta^{j)} T_{kl l_1 \dots l_m}^{m \beta} = 0.$$

We covariantly differentiate (18) along  $x^l$ . Because  $F_i^h$  and  $\lambda^h$  are covariantly constant, after an application (12) we obtain

$$(19) \quad A_{l_1 j k}^{h i} \Omega_{l_2 l_3 \dots l_m l} + A_{l_2 j k}^{h i} \Omega_{l_1 l_3 \dots l_m l} + \dots + A_{l_m j k}^{h i} \Omega_{l_1 l_2 \dots l_{m-1} l} = 0,$$

where

$$(20) \quad A_{l j k}^{h i} \stackrel{\text{def}}{=} \lambda^{(h} R_{l j k}^{i)} + \lambda^\alpha F_\alpha^{(h} F_\beta^{i)} R_{l j k}^\beta.$$

If  $A_{l j k}^{h i} \neq 0$  then exist a tensor  $\varepsilon_{h i}^{j k}$  such that  $A_l = \varepsilon_{h i}^{j k} A_{l j k}^{h i} \neq 0$ . Contracting (19) with  $\varepsilon_{h i}^{j k}$  and  $\varepsilon^l$  from (17) we obtain the term (13).  $A_l = 0$  holds according to

Lemma 2, which is a contradiction. Thus  $A_{ljk}^{hi} = 0$  it means with respect to (20) that

$$\lambda^{(h} R_{ljk}^{i)} + \lambda^\alpha F_\alpha^{(h} F_\beta^{i)} R_{ljk}^\beta = 0.$$

Hence it clearly follows from  $\lambda^h \neq 0$  that  $R_{ljk}^h = 0$ , i.e. the space  $K_n^m$  is flat and according to Lemma 1 the space  $\bar{K}_n$  has constant holomorphic curvature.  $\square$

According to Theorems 3, 4 and 5 it follows that generalized recurrent spaces  $A_n$  from these Theorems, which are not flat, do not admit the mentioned nontrivial holomorphically projective mappings onto Kählerian spaces.

These results are a generalization of results by T. Sakaguchi, J. Mikeš and V. V. Domashev, which were done for holomorphically projective mappings of symmetric, recurrent and semisymmetric Kählerian spaces [2, 14, 16, 21].

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BASIC EDUCATION COLLEGE,  
BASRAH UNIVERSITY, IRAQ,  
*E-mail*: raadjaka@yahoo.com

DEPARTMENT OF ALGEBRA AND GEOMETRY  
FACULTY OF SCIENCE, PALACKY UNIVERSITY  
TOMKOVA 40, 779 00 OLOMOUC, CZECH REPUBLIC  
*E-mail*: skodova@inf.upol.cz

DEPARTMENT OF ALGEBRA AND GEOMETRY  
FACULTY OF SCIENCE, PALACKY UNIVERSITY  
TOMKOVA 40, 779 00 OLOMOUC, CZECH REPUBLIC  
*E-mail*: mikes@inf.upol.cz