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Maximally homogeneous nondegenerate CR manifolds

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Abstract. We prove that a maximally homogeneous nondegenerate CR manifold is standard. If it is compact, then it is a real projective manifold.

Introduction

An *almost CR manifold* is a smooth paracompact manifold M endowed with a *partial almost-complex structure* (H, J), where H is a subbundle of its (real) tangent bundle TM (the *analytic tangent bundle of* M), and $J : H \to H$ a smooth fiber preserving bundle isomorphism (the *almost-complex structure of* M), with $J^2 = -\mathrm{id}$ and

$$[X, Y] - [JX, JY] \in \mathscr{C}^\infty(M, H) \quad \forall X, Y \in \mathscr{C}^\infty(M, H).$$

Here $\mathscr{C}^\infty(M, H)$ denotes the space of smooth sections of H. The real rank r of H is even; $n = r/2$ is the *CR-dimension* of the manifold M and $k = \dim M - 2n$ its *CR-codimension*. The almost CR manifold M is said to be *nondegenerate at* $x \in M$ if for every $X \in \mathscr{C}^\infty(M, H)$ with $X_x \neq 0$ there exists $Y \in \mathscr{C}^\infty(M, H)$ such that $[X, Y]_x \notin H_x$, and *nondegenerate* if M is such at every point.

To the (nonintegrable) distribution $\mathscr{D}_{-1} = \mathscr{C}^\infty(M, H)$, we associate a sequence of linear spaces of smooth vector fields $\{0\} = \mathscr{D}_0 \subset \mathscr{D}_{-1} \subset \cdots \subset \mathscr{D}_{-p} \subset \cdots$, with $\mathscr{D}_{-p} = \mathscr{D}_{-p+1} + [\mathscr{D}_{-1}, \mathscr{D}_{-p+1}]$ if $p \geqslant 2$. This gives, at each point $x \in M$, a graded Lie algebra $\mathfrak{m}(x) = \bigoplus_{p<0} \mathfrak{g}_p(x)$, where $\mathfrak{g}_p(x) = (\mathscr{D}_p)_x/(\mathscr{D}_{p+1})_x$ for $p < 0$ and the Lie commutator in $\mathfrak{m}(x)$ is obtained from the commutator of vector fields by passing to the quotients (see e.g. [11]). The almost-complex structure J of M defines a complex structure J_x on $\mathfrak{g}_{-1}(x)$, such that $[J_x X, J_x Y] = [X, Y]$ for every $X, Y \in \mathfrak{g}_{-1}(x)$.

We say that M is *of finite type* (in the sense of Bloom–Graham) at $x \in M$ if $\dim_{\mathbb{R}} \mathfrak{m}(x) = \dim_{\mathbb{R}} M$, and *regular of type* $\mathfrak{m}$ if moreover all $\mathfrak{m}(x)$ for $x \in M$ are isomorphic to a fixed pseudocomplex graded Lie algebra $\mathfrak{m} = \bigoplus_{p<0} \mathfrak{g}_p$. Pseudocomplex means that a complex structure $J_{\mathfrak{m}}$ is given on $\mathfrak{g}_{-1}$, such that $[J_{\mathfrak{m}} X, J_{\mathfrak{m}} Y] = [X, Y]$ for every $X, Y \in \mathfrak{g}_{-1}$.

In [11], N. Tanaka constructed the maximal transitive pseudocomplex prolongation $\mathfrak{g} = \mathfrak{g}(\mathfrak{m}) = \bigoplus_{p \in \mathbb{Z}} \mathfrak{g}_p$ of $\mathfrak{m}$, which is unique modulo isomorphisms (and that we called in [5] the *Levi–Tanaka algebra* of $\mathfrak{m}$). The elements of $\mathfrak{g}_0$ are required to define 0-degree derivations of $\mathfrak{m}$ which commute with $J_{\mathfrak{m}}$ on $\mathfrak{g}_{-1}$. He proved that $\dim_{\mathbb{R}} \mathfrak{g} < \infty$ if and only if $\mathfrak{m}$ is nondegenerate (i.e. if $[X, \mathfrak{g}_{-1}] \neq 0$ when $X \in \mathfrak{g}_{-1}$ is $\neq 0$). Under this assumption, he showed that for an almost CR manifold M of type $\mathfrak{m}$, the group of CR automorphisms of M is a Lie group of dimension less than or equal to the dimension of the Levi-Tanaka algebra $\mathfrak{g}$ of $\mathfrak{m}$.

We call a nondegenerate CR manifold M of type $\mathfrak{m}$ *maximally homogeneous* if its group of CR automorphisms has dimension equal to $\dim_{\mathbb{R}} \mathfrak{g}(\mathfrak{m})$.

In [5] we associated to every finite-dimensional Levi–Tanaka algebra $\mathfrak{g} = \bigoplus_{p \in \mathbb{Z}} \mathfrak{g}_p$ a homogeneous CR manifold $S = S(\mathfrak{g})$ that we called *standard*: $S = G/G_+$, where G is the connected and simply connected Lie group having Lie algebra $\mathfrak{g}$ and G_+ the (closed) analytic Lie subgroup of G with Lie algebra $\mathfrak{g}_+ = \bigoplus_{p \geqslant 0} \mathfrak{g}_p$. In particular, S is simply connected and maximally homogeneous.

In this paper we prove that a maximally homogeneous nondegenerate CR manifold M, regular of type $\mathfrak{m}$, is CR-diffeomorphic to the standard one associated to the prolongation $\mathfrak{g}$ of $\mathfrak{m}$.

The characterization of these manifolds extends the results for CR manifolds of hypersurface type (i.e. with CR-codimension equal to one) obtained by Yamaguchi in [13] to the case of CR manifolds of arbitrary CR-codimension, and is a CR analogue of classical results on manifolds with maximal groups of isometries in Riemannian geometry (cf. e.g. note 10 to [3], or [4]).

We shall consider the compact case first, and next derive the general result using the Mostow fibration obtained in [8]. Note that the classification of semisimple Levi–Tanaka algebras of [6] gives because of [8] a classification of the compact standard CR manifolds and therefore, by the result proved here, of all compact maximally homogeneous nondegenerate CR manifolds.

1 Definitions and notation

By a graded Lie algebra we mean a $\mathbb{Z}$-graded Lie algebra over $\mathbb{R}$ with $\dim_{\mathbb{R}} \mathfrak{g}_p < \infty$ for all $p \in \mathbb{Z}$. We say that $\mathfrak{g}$ is:

- *fundamental* if $\mathfrak{g}_p = 0$ for $p \geqslant 0$ and $\mathfrak{g}_{p-1} = [\mathfrak{g}_p, \mathfrak{g}_{-1}]$ for all $p \leqslant -1$;
- *nondegenerate* if $[X, \mathfrak{g}_{-1}] \neq 0$ for all $0 \neq X \in \mathfrak{g}_{-1}$;
- *characteristic* if the center of $\mathfrak{g}_0$ contains a characteristic element E, i.e.

$$[E, X] = pX \qquad \text{for all} \qquad X \in \mathfrak{g}_p, p \in \mathbb{Z};$$

- *transitive* if $[X, \mathfrak{g}_{-1}] \neq 0$ for all $0 \neq X \in \bigoplus_{p \geqslant 0} \mathfrak{g}_p$;
- *pseudocomplex* if an element $J \in \mathrm{Hom}_{\mathbb{R}}(\mathfrak{g}_{-1}, \mathfrak{g}_{-1})$ is given such that

$$J^2 = -\mathrm{id}_{\mathfrak{g}_{-1}} \quad \text{and} \quad \begin{cases} [JX, JY] = [X, Y] & \forall X, Y \in \mathfrak{g}_{-1}, \\ [A, JX] = J[A, X] & \forall A \in \mathfrak{g}_0, \ \forall X \in \mathfrak{g}_{-1}. \end{cases}$$

A transitive pseudocomplex graded real Lie algebra $\mathfrak{g} = \bigoplus_{p\in\mathbb{Z}} \mathfrak{g}_p$ is called a *Levi–Tanaka algebra* if its subalgebra $\mathfrak{m}(\mathfrak{g}) = \bigoplus_{p<0} \mathfrak{g}_p$ is fundamental, and $\mathfrak{g}$ is maximal in the class of transitive pseudocomplex graded real Lie algebras $\mathfrak{a}$ with $\mathfrak{m}(\mathfrak{a}) = \bigoplus_{p<0} \mathfrak{a}_p = \mathfrak{m}(\mathfrak{g})$. Note that Levi–Tanaka algebras and semisimple graded Lie algebras are characteristic.

Given a characteristic graded Lie algebra $\mathfrak{a} = \bigoplus_{p\in\mathbb{Z}} \mathfrak{a}_p$, we denote by $G = G(\mathfrak{a})$ the connected and simply connected Lie group having Lie algebra $\mathfrak{a}$, and by $G_+ = G_+(\mathfrak{a})$ the analytic subgroup of G having Lie algebra $\mathfrak{a}_+ = \bigoplus_{p\geqslant 0} \mathfrak{a}_p$. Then we define the *standard manifold* $S = S(\mathfrak{a})$ associated to $\mathfrak{a}$ to be the homogeneous space

$$S = G/G_+.$$

Note that G_+ is closed in G since it is the connected component of the identity of the normalizer $N_G(\mathfrak{a}_+)$ of $\mathfrak{a}_+$ in G.

For simplicity we shall assume that all manifolds considered in the following are connected.

2 Compact standard CR manifolds

In [8] we showed that the standard CR manifold $S = G/G_+$, associated to a finite-dimensional Levi–Tanaka algebra $\mathfrak{g} = \bigoplus_{p\in\mathbb{Z}} \mathfrak{g}_p$, is compact if and only if $\mathfrak{g}$ is semisimple. For the application to the general case, we need however to consider in this section a slightly more general situation. Namely, we shall assume that $\mathfrak{g}$ is a finite-dimensional semisimple pseudocomplex fundamental graded Lie algebra (we drop the requirement that $\mathfrak{g}$ is nondegenerate). Also in this case the homogeneous manifold G/G_+ is compact.

Denote by G' the adjoint group of the Lie algebra $\mathfrak{g}$ and by G'_+ the analytic subgroup of G' generated by $\mathfrak{g}_+ = \bigoplus_{p\geqslant 0} \mathfrak{g}_p$. We consider the homogeneous space $S' = G'/G'_+$ endowed with its natural CR structure: the covering homomorphism $G \to G'$ of the connected and simply connected Lie group G having Lie algebra $\mathfrak{g}$ onto G' defines a CR covering map $S \to S'$. We rehearse the construction of the CR structure of S' (see [8] for the analogous discussion of the CR structure of S). We shall use the same letter J for the complex structure on $\mathfrak{g}_{-1}$ and the partial almost-complex structure of S'. Let $\pi : G' \to S'$ be the projection onto the quotient and $o = \pi(e) = G'_+$ the image of the identity e of G'. With $\mathfrak{g}_{(-1)} = \bigoplus_{p\geqslant -1} \mathfrak{g}_p$, we set:

$$H_o = \pi_*(\mathfrak{g}_{(-1)}), \tag{1}$$

$$J\pi_*(X) = \pi_*(JX_{-1}) \quad \forall X = \sum_{p\geqslant -1} X_p \in \mathfrak{g}_{(-1)}, X_p \in \mathfrak{g}_p, \tag{2}$$

and, for $x = \pi(g) = gG'_+ \in S'$, with $g \in G'$,

$$H_x = g_* H_o, \tag{3}$$

$$J(g_*(X)) = g_*(JX) \quad \forall X \in H_o. \tag{4}$$

This is a consistent definition: indeed, for $g \in G'_+$, we have:

$$\mathrm{Ad}_{G'}\, g(\mathfrak{g}_{(-1)}) \subset \mathfrak{g}_{(-1)}, \tag{5}$$

$$\pi_*(\mathrm{Ad}_{G'}\, g(JX)) = \pi_*(JX) \quad \forall X \in \mathfrak{g}_{-1}. \tag{6}$$

Using Corollary 2.5 of [6], we fix a minimally compact Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{g}$ contained in $\mathfrak{g}_0$. Let $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ be a Cartan decomposition of $\mathfrak{g}$, where $\mathfrak{k}$ is a maximal compact subalgebra of $\mathfrak{g}$, and $\mathfrak{h} \cap \mathfrak{p} = \mathfrak{h}_{\mathfrak{p}}$ a maximal Abelian subalgebra contained in $\mathfrak{p}$. Denote by $\Sigma \subset \mathfrak{h}_{\mathfrak{p}}^*$ the system of (restricted) roots associated to the pair $(\mathfrak{g}, \mathfrak{h}_{\mathfrak{p}})$. It is possible to fix an order of Σ such that

$$\mathfrak{n}^+ = \bigoplus_{\substack{\lambda \in \Sigma \\ \lambda > 0}} \mathfrak{g}^\lambda \supset \bigoplus_{p>0} \mathfrak{g}_p, \tag{7}$$

where $\mathfrak{g}^\lambda = \{X \in \mathfrak{g} \mid [A, X] = \lambda(A)X \ \forall A \in \mathfrak{h}_{\mathfrak{p}}\}$. Then we have the Iwasawa decomposition

$$\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{h}_{\mathfrak{p}} \oplus \mathfrak{n}^+. \tag{8}$$

Let N^+ be the analytic subgroup of G' generated by $\mathfrak{n}^+$. It acts on S' by restriction of the action of G'. Note that the N^+ orbit of $o = G'_+$ reduces to $\{o\}$. We have the following:

Lemma 2.1. *The manifold S' is a finite union of N^+ orbits, which are topologically Euclidean. There is a single open N^+ orbit, which is dense in S'; all other N^+ orbits have a* CR*-dimension which is strictly smaller than that of S'. Moreover, their tangent spaces intersect the analytic tangent space H of S' in J-invariant subspaces.*

Proof. We use the generalized Bruhat decomposition of G' (see Theorem 8 of [10]). It yields a decomposition of S' into N^+ orbits $V_w = N^+ w G'_+$ for w belonging to a finite subset of the normalizer $N_K(\mathfrak{h}_{\mathfrak{p}})$ of the subalgebra $\mathfrak{h}_{\mathfrak{p}}$ in the analytic subgroup K of G' generated by $\mathfrak{k}$; these orbits are topologically Euclidean and give a cell decomposition of S'. Exactly one of them is open and dense (see e.g. Proposition 1.2.4.10 of [12]). Note that an orbit having the same CR-dimension as S' is open, because S' is of finite type.

We consider now an orbit V_w, for a fixed $w \in N_K(\mathfrak{h}_{\mathfrak{p}})$, and we show that the intersection between the tangent space $T_{\pi(w)}V_w$ to V_w at $\pi(w)$ and $H_{\pi(w)}$ is J-invariant. Notice that

$$T_{\pi(w)}V_w \cap H_{\pi(w)} \cong (\mathfrak{g}_{(-1)} \cap \mathrm{Ad}_{G'}\, w^{-1}(\mathfrak{n}^+))/(\mathfrak{g}_+ \cap \mathrm{Ad}_{G'}\, w^{-1}(\mathfrak{n}^+)). \tag{9}$$

Because $\mathfrak{g}$ is semisimple, the almost-complex structure J on $\mathfrak{g}_{-1}$ is the restriction of an inner derivation $\mathrm{ad}_{\mathfrak{g}}\tilde{J}$ for a $\tilde{J} \in \mathfrak{g}_0$; in fact, $\tilde{J} \in \mathfrak{k} \cap \mathfrak{h}$ (cf. Theorem 2.4 and Corollary 2.5 of [6]). We need to prove that

$$X \in \mathfrak{g}_{(-1)}, \mathrm{Ad}_{G'}\, w(X) \in \mathfrak{n}^+ \Rightarrow \mathrm{Ad}_{G'}\, w([\tilde{J}, X]) \in \mathfrak{n}^+. \tag{10}$$

We have

$$0 = \mathrm{Ad}_{G'}\, w([\tilde{J}, \mathfrak{h}_\mathfrak{p}]) = [\mathrm{Ad}_{G'}\, w(\tilde{J}), \mathrm{Ad}_{G'}\, w(\mathfrak{h}_\mathfrak{p})] = [\mathrm{Ad}_{G'}\, w(\tilde{J}), \mathfrak{h}_\mathfrak{p}], \tag{11}$$

and therefore $\mathrm{Ad}_{G'}\, w(\tilde{J})$ belongs to the centralizer $\mathfrak{c}_\mathfrak{k}(\mathfrak{h}_\mathfrak{p})$ of $\mathfrak{h}_\mathfrak{p}$ in $\mathfrak{k}$. Because $\mathfrak{c}_\mathfrak{k}(\mathfrak{h}_\mathfrak{p}) \oplus \mathfrak{h}_\mathfrak{p} = \mathfrak{g}^0$, from (7) we obtain $[\mathfrak{c}_\mathfrak{k}(\mathfrak{h}_\mathfrak{p}), \mathfrak{n}^+] = \mathfrak{n}^+$, and therefore

$$\mathrm{Ad}_{G'}\, w([\tilde{J}, X]) = [\mathrm{Ad}_{G'}\, w(\tilde{J}), \mathrm{Ad}_{G'}\, w(X)] \in \mathfrak{n}^+. \tag{12}$$

By Lemma 2.1 the manifold S' is the union of an open dense simply connected subset and of finitely many locally closed submanifolds of S' of codimension greater than one. Then we obtain (see e.g. Proposition 12.4 of Chap. VII of [2]):

Proposition 2.2. *The manifold S', constructed above, is simply connected and therefore diffeomorphic to the standard one $S = G/G_+$.*

Remark 2.3. The normalizer $N_{G'}(\mathfrak{g}_+)$ of $\mathfrak{g}_+$ in G' is connected, and thus coincides with G'_+. In particular, $\mathfrak{g}_+$ is equal to its normalizer in $\mathfrak{g}$ (the action of G' on S' is *asystatic*, according to the definition of [9]) and is an algebraic Lie subalgebra of $\mathfrak{g}$.

For standard CR manifolds associated to Levi–Tanaka algebras we obtain the following

Corollary 2.4. *Let $\mathfrak{g} = \bigoplus_{p \in \mathbb{Z}} \mathfrak{g}_p$ be a semisimple Levi–Tanaka algebra and G the connected and simply connected Lie group having Lie algebra $\mathfrak{g}$. Then every connected G-homogeneous almost* CR *manifold regular of type $\mathfrak{m} = \bigoplus_{p<0} \mathfrak{g}_p$ is* CR*-diffeomorphic to the standard* CR *manifold. In particular, the almost* CR *manifold S', constructed as above starting now from a Levi–Tanaka algebra, is* CR*-diffeomorphic to the standard* CR *manifold S.*

Proof. By [8] every connected $\mathfrak{m}$-regular G-homogeneous almost *CR* manifold M is covered by the standard one. Then Lemma 2.1 provides a finite cell decomposition of M with no cells of dimension $\dim_\mathbb{R} M - 1$. As in Proposition 2.2, we conclude that M is simply connected. This completes the proof of the corollary.

Now we have the following

Theorem 2.5. *The group of* CR*-automorphisms of a compact standard* CR *manifold $S = G/G_+$ associated to a Levi–Tanaka algebra $\mathfrak{g}$ has trivial center and its connected component of the identity is isomorphic to the adjoint group G' of $\mathfrak{g}$.*

There exists a G'-equivariant (and G-equivariant) projective embedding of every compact standard CR *manifold $S = G/G_+ \cong S' = G'/G'_+$.*

Proof. The first statement of the theorem follows from Theorem 4.4 of [5].

The manifold S' is real projective by Theorem 10 of [10]. To describe the G'-equivariant complex projective embedding of S', we follow the construction in §4.3 of [5]. Let $\hat{\mathfrak{g}}$ be the complexification of the semisimple Levi–Tanaka algebra $\mathfrak{g}$ and let $\mathfrak{q} = \bigoplus_{p \geqslant -1} \mathfrak{q}_p$ denote the complex Lie subalgebra of $\hat{\mathfrak{g}}$ defined by

$$\mathfrak{q} = \begin{cases} \hat{\mathfrak{g}}_p = \mathbb{C} \otimes_{\mathbb{R}} \mathfrak{g}_p & \text{if } p \geqslant 0, \\ \{X + \sqrt{-1}JX \mid X \in \mathfrak{g}_{-1}\} & \text{if } p = -1. \end{cases} \tag{13}$$

Let $\hat{G}$ be the adjoint group of $\hat{\mathfrak{g}}$ and Q the analytic subgroup of $\hat{G}$ generated by $\mathfrak{q}$. Then Q is parabolic and $\mathscr{F} = \hat{G}/Q$ is a flag manifold; in particular it is complex projective. We have a natural inclusion $G' \hookrightarrow \hat{G}$, yielding the CR-immersion

$$S \cong S' = G'/G'_+ \xrightarrow{\varphi} G'/(G' \cap Q) \subset \mathscr{F}. \tag{14}$$

Then φ is a covering map and, by Proposition 2.2, is a CR diffeomorphism; hence the map $S' \to \mathscr{F}$ is a *generic* CR embedding (i.e. with $\dim S' - \text{CR-}\dim S' = \dim_{\mathbb{C}} \mathscr{F}$).

Remark 2.6. We already noted that G' is the connected component of the identity in the group of CR-automorphisms of $S \cong S'$. In general this is not connected but it has a finite number of connected components (it follows from [1], Theorem 4.1 of Chapter 2). For example, if S is the CR quadric associated to $\mathfrak{g} = \mathfrak{su}(p+1, p+1)$, then the group of CR-automorphisms of S is isomorphic to the projective conformal group $\mathbb{P}\mathrm{CU}(p+1, p+1)$, which has two connected components; the connected component of the identity is isomorphic to $\mathbb{P}\mathrm{SU}(p+1, p+1)$, i.e. to the quotient of $\mathrm{SU}(p+1, p+1)$ by its center.

3 The general case

We have the following:

Theorem 3.1. *A maximally homogeneous nondegenerate* CR *manifold, regular of type* $\mathfrak{m} = \bigoplus_{p<0} \mathfrak{g}_p$, *is* CR *diffeomorphic to the standard* CR *manifold S associated to the maximal pseudocomplex prolongation* $\mathfrak{g}$ *of* $\mathfrak{m}$.

In particular, the group of CR-*automorphisms of S has trivial center and its connected component of the identity is isomorphic to the adjoint group of* $\mathfrak{g}$.

Proof. Using the construction of the tower of principal fibrations given in [11], we obtain that a maximally homogeneous nondegenerate CR manifold M is homogeneous, and hence regular.

The group of its CR-automorphisms is a Lie group having Lie algebra isomorphic to the maximal transitive pseudocomplex prolongation $\mathfrak{g}$ of its type $\mathfrak{m}$ (it follows from Theorem 10.2 of [7]).

By Theorem 5.2 of [8], we have a CR Mostow fibration of M over a compact CR manifold B, which admits a CR universal covering $\tilde{B} \to B$ by the Cartesian product $\tilde{B}$ of two compact manifolds, one being a nondegenerate standard CR manifold and the other one a Hermitian symmetric space. Note that we have a sequence of covering maps $\tilde{B} \to B \to B'$, where B' is constructed as in section 2 from a pseudocomplex fundamental graded Levi subalgebra of $\mathfrak{g}$ (see Theorem 1.1 of [8]). By Proposition 2.2 the covering map $\tilde{B} \to B'$ is injective and therefore B is homeomorphic to $\tilde{B}$, and thus simply connected. Then we have obtained that M is simply connected and therefore standard.

The last statement follows then from Theorem 4.4 of [5].

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