

**A THEOREM ON SPHERICAL QUADRILATERALS  
AND SPHERICAL CONICS**

**By**

**William Rowan Hamilton**

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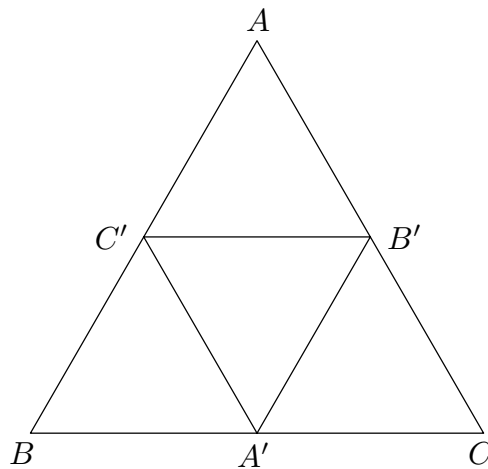
*A Theorem on Spherical Quadrilaterals and Spherical Conics.*

By Sir WILLIAM R. HAMILTON.

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The following notice, by the President, Sir William R. Hamilton, of a theorem derived from his Researches on Quaternions, was read.



Let  $AC'A'B'$  be called a *spherical parallelogram*, if  $A'$ ,  $B'$ ,  $C'$  bisect the sides  $BC$ ,  $CA$ ,  $AB$  of a spherical triangle  $ABC$ ; and let it be said that the corner  $A$  of the triangle is the point which *completes the parallelogram* when  $A'B'$  and  $A'C'$  are given as two adjacent sides thereof.

Take any spherical quadrilateral,  $KLMN$ , and any point on the same spheric surface,  $P$ ; draw the four arcs  $PK$ ,  $PL$ ,  $PM$ ,  $PN$ , and complete, in four points,  $K'$ ,  $L'$ ,  $M'$ ,  $N'$ , the four spherical parallelograms, of which the given pairs of adjacent sides are  $PK$ ,  $PL$ ;  $PL$ ,  $PM$ ;  $PM$ ,  $PN$ ;  $PN$ ,  $PK$ . Then the four new points,  $K'$ ,  $L'$ ,  $M'$ ,  $N'$ , form a new spherical quadrilateral, such that its four sides  $K'L'$ ,  $L'M'$ ,  $M'N'$ ,  $N'K'$ , touch a certain spherical conic, having the poles of the diagonals  $KM$ ,  $LN$  of the old quadrilateral for its foci.

This theorem was stated to follow as an easy corollary from what Sir William Hamilton had already communicated to the Academy respecting quaternions.