QUATERNION PROOF OF A THEOREM OF RECIPROCITY OF CURVES IN SPACE

By

William Rowan Hamilton

(British Association Report, 1862, Part II, p. 4.)

Edited by David R. Wilkins

2000

Quaternion Proof of a Theorem of Reciprocity of Curves in Space. By Sir William Rowan Hamilton, *LL.D., &c.*

[*Report of the Thirty-Second Meeting of the British Association for the Advancement of Science; held at Cambridge in October 1862.*
(John Murray, London, 1863), Part II, p. 4.]

Let ϕ and ψ be any two vector functions of a scalar variable, and ϕ', ψ', ϕ'', ψ'' their derived functions, of the first and second orders. Then each of the two systems of equations, in which c is a scalar constant,

$$(1)\ \ldots\quad \mathrm{S}\phi\psi = c,\quad \mathrm{S}\phi'\psi = 0,\quad \mathrm{S}\phi''\psi = 0,$$

$$(2)\ \ldots\quad \mathrm{S}\psi\phi = c,\quad \mathrm{S}\psi'\phi = 0,\quad \mathrm{S}\psi''\phi = 0,$$

or each of the two vector expressions,

$$(3)\ \ldots\quad \psi = \frac{cV\phi'\phi''}{S\phi\phi'\phi''},\quad (4)\ \ldots\quad \phi = \frac{cV\psi'\psi''}{S\psi\psi'\psi''},$$

includes the other.

If then, from any assumed origin, there be drawn lines to represent the reciprocals of the perpendiculars from that point on the osculating planes to a first curve of double curvature, those lines will terminate on a second curve, from which we can *return* to the first by a precisely similar process of construction.

And instead of thus taking the *reciprocal* of a *curve* with respect to a *sphere*, we may take it with respect to *any surface* of the *second order*, as is probably well known to geometers, although the author was lately led to perceive it for himself by the very simple *analysis* given above.