Introduction

Paul Erdős was always interested in infinity. One of his earliest results is an infinite analogue of (the then very recent) Menger's theorem (which was included in a classical book of his teacher Denes König). Two out of his earliest three combinatorial papers are devoted to infinite graphs. According to his personal recollections, Erdős always had an interest in "large cardinals" although his earliest work on this subject are joint papers with A. Tarski from the end of thirties. These interests evolved over the years into the Giant Triple Paper, with the Partition Calculus forming a field rightly called here Erdősian Set Theory.

We wish to thank András Hajnal for a beautiful paper which perhaps best captures the special style and spirit of Erdős' mathematics. We solicited another two survey papers as well. An extensive survey was written by Peter Cameron on the seemingly simple subject of the infinite random graph, which describes the surprising discovery of Rado and Erdős-Renyi finding many new fascinating connections and applications. The paper by Peter Komjáth deals with another (this time more geometrical) aspect of Erdősian set theory. In addition, the research articles by Shelah and Kříž complement the broad scope of today's set theory research, while the paper of Aharoni looks at another pre-war Erdős conjecture. And here are the masters own words:

I have nearly 50 joint papers with Hajnal on set theory and many with Rado and Fodor and many triple papers and I only state a few samples of our results. The first was my result with Dushnik and Miller.

Let $m \geq \aleph_0$. Then

$$m \to (m, \aleph_0)^2_2 .$$

I use the arrow notation invented by R. Rado — in human language: If one colors the pairs of a set of power $m \geq \aleph_0$ by two colors either in color 1 there is a complete graph of power m or in color 2 an infinite complete graph. Hajnal, Rado and I nearly completely settled $m \to (n, q)^2_2$ but the results are very technical and can be found in our joint triple paper and in our book. In one of our joint papers, Hajnal and I proved that a graph G of chromatic number $\aleph_1$ contains all finite bipartite graphs and with Shelah and Hajnal we proved that it contains all sufficiently large odd cycles (Hajnal and Komjáth have sharper results).

Hajnal and I have quite a few results on property B. A family of sets $\{A_\alpha\}$ has property B if there is a set C which meets each of the sets A_α

and contains none of them. This definition is due to Miller. It is now more customary to call the family two chromatic. Here is a sample of our many results: Let $\{A_\alpha\}$ be a family of $\aleph_k$ countable sets with $|A_\alpha \cap A_\beta| \leq n$. Then there is a set S which meets each A_α in a set of size $(k+1)n+1$. The result is best possible. If $k \geq \omega$ then there is a set for which $|S \cap A_\alpha| < \aleph_0$. We have to assume the generalized hypothesis of the continuum.

Todorčevič proved with $c = \aleph_1$ that one can color the edges of the complete graph on $|S| = \aleph_1$ with $\aleph_1$ colors so that every $S_1 \subset S$, $|S_1| = \aleph_1$, contains all colors.

We wrote two papers on solved and unsolved problems in set theory. Most of them have been superseded in many cases because undecidability raised its ugly head (according to many: its pretty head). Here is a problem where, as far as we know, no progress has been made. One can divide the triples of a set of power $2^{2^{\aleph_0}}$ into t classes so that every set of power $\aleph_1$ contains a triple of both classes. On the other hand if we divide the triples of a set of power $(2^{2^{\aleph_0}})^+$ into two classes there is always a set of size $\aleph_1$ all whose triples are in the same class. If $S = 2^{2^{\aleph_0}}$ can we divide the triples into two classes so that every subset of size $\aleph_1$ should contain a $K(4)$ of both classes (or more generally a homogeneous subset of size $\aleph_0$?) I offer 500 dollars for clearing up this problem.

Erdős-Galvin-Hajnal problem. Let G have chromatic number $\aleph_1$. Can one color the edges by 2 (or $\aleph_0$, or $\aleph_1$) colors so that if we divide the vertices into $\aleph_0$ classes there always is a class which contains all the colors? Todorčevič proved this if G is the complete graph of $\aleph_1$ vertices — in the general case Hajnal and Komjáth have some results.