Alon, Noga; Erdős, Paul; Holzman, Ron; Krivelevich, Michael
On k-saturated graphs with restrictions on the degrees. (In English)
J. Graph Theory 23, No.1, 1-20 (1996). [0364-9024]

A graph G is called k-saturated if it is K_k-free, but the addition of any edge produces a K_k (where K_k is the complete graph of order k). There is an old result that if G has order n and is k- saturated, then the number of edges in G is at least $(k-2)n - \binom{k-1}{2}$. However, the extremal examples contain each vertex of degree $n-1$. This paper defines $F_k(n, D)$ to be the minimal number of edges in a k-saturated graph of order n and maximum degree at most D. The case $k = 3$ has been studied by *Z. Füredi* and *Á. Seress* [J. Graph Theory 18, No. 1, 11-24 (1994; Zbl 787.05054)].
In this paper it is shown that $F_4(n, D) = 4n - 15$ for $n > n_0$ and $\lfloor (2n - 1)/3 \rfloor \leq D \leq n - 2$. Also, it is shown that $\lim_{n \to \infty} F_k(n, cn)$ exists for all $0 < c \leq 1$, except maybe for some values of c contained in a sequence $c_i \to 0$. For sufficiently large n, they also construct some k- saturated graphs of order n with maximal degree at most $2k\sqrt{n}$, significantly improving earlier results.
C.Jagger (Cambridge)

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