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Equal distance sums in the plane. (In English)

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Let $X_n = \{x_1, \ldots, x_n\}$ be a set of n distinct points in the plane; and let S_i be the sum of (Euclidean) distances from the point x_i to the other points in X_n. X_n is called an equisum set if $S_1 = \cdots = S_n$.
It is obvious that X_3 is an equisum set iff X_3 is the vertex set of a regular triangle. In this paper is simply shown that X_4 is an equisum set iff its points are the corners of a rectangle. The authors investigate all about the five-point set X_5. Here the quasi-convexity and the bilaterally symmetry play a role.
X_n is called quasi-convex if its points are the vertices of a strictly convex n-gon, and bilaterally-symmetric if X_n is identical to the set obtained by rotating its points 180° around some line.
All equisum sets X_n are quasi-convex.
Some interesting results:
– If X_5 is a bilaterally symmetric equisum set, then it has exactly 2, 5 or 6 different interpoint distances.
– If X_5 is an equisum set, but not bilaterally symmetric, then it has exactly 8, 9, or 10 different interpoint distances.
In the cases that the number of interpoint distances is 6, 9 or 10 there are infinitely many dissimilar equisum sets X_5.
This paper is related to papers by *P. Erdős* and *P. Fishburn* [Discrete Appl. Math. 60, No. 1-3, 149-158 (1995; Zbl 831.52009)], by *V. Klee* and *St. Wagon* [‘Old and new unsolved problems in plane geometry and number theory’ (1991; Zbl 784.51002)] and others.

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Classification:
52C10 Erdoes problems and related topics of discrete geometry
52A40 Geometric inequalities, etc. (convex geometry)
51M16 Inequalities and extremum problems (geometry)

Keywords:
Erdős problem; minimal number of points; equisum set; distinct distances in finite point sets