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Erdős, Paul; Fishburn, Peter
Convex nonagons with five intervertex distances. (In English)
Geom. Dedicata 60, No.3, 317-332 (1996). [0046-5755]

The authors prove that the vertices of a convex 9-gon in the euclidean plane determine exactly five distinct distances if and only if they are a subset of the vertices of a regular 10-gon or a regular 11-gon, and conjecture that the vertices of a convex $2m+1$-gon determine exactly $m+1$ distance ($m \geq 3$) if and only if they are a subset of the vertices of a regular $2m+2$- or $2m+3$-gon. This may be seen as one step below the extremal situation of Altman's theorem which states that the vertices of a convex n-gon determine at least $\lfloor \frac{1}{n} n \rfloor$ distinct distances, with equality if and only if the n-gon is regular, or n is even and it is a regular $n+1$-gon minus one point.
The main reason for the study of point sets with few distinct distances is the famous Erdős conjecture on the minimal number of distinct distances determined by n points in the plane, which is believed to be $\Theta(\frac{n}{\sqrt{\log n}})$ as can be reached by sections of the triangular lattice.

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Classification:
52C10 Erdoes problems and related topics of discrete geometry
52C05 Lattices and convex bodies in 2 dimensions

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intervertex distances; convex polygons; Erdős conjecture