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Tournaments that share several common moments with their complements. (In English)

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The k-th moment of a tournament T is the sum of the k-th powers of its scores, that is, $M_k(T) = \sum_{i=1}^n t_i^k$. A tournament T and its complement T^c are said to share the k-th moment if $M_k(T) = M_k(T^c)$. We define the common moment set of T and T^c as $P = \{k \in \mathbb{N} \mid M_k(T) = M_k(T^c)\}$. Some tournaments have self complementary score sequences, which forces $P = \mathbb{N}$. But, when the sequence is not self complementary, P is a finite subset of $\mathbb{N}$. For any tournament, $1, 2 \in P$. In fact, $P = \{1, 2, \dots, 2p\} \cup A$ where $A \subset \{2p+1, 2p+2, \dots\}$ with $2p+1, 2p+2 \notin A$. For every even integer $2p$, we explicitly construct a tournament which shares the first $2p$ common moments with its complement, and furthermore, we seek the smallest such tournament. This can be achieved with $cp^2 \ln p$ vertices. Paul Erdős asked whether any tournament and its complement yield a nonempty set A. For a long time we could not find any example with A nonempty. In this paper, we now show that nonempty sets A can occur provided they have a certain low "initial density". Furthermore, we characterize the sets A that can occur and thus we also characterize sets P which can be the common moment set of T and T^c. We also give explicit examples of tournaments attaining P for a few small sets P.

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05C20 Directed graphs (digraphs)
11B75 Combinatorial number theory

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moment; tournament; common moment set; complementary score sequences