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Turán-Ramsey theorems and K_p-independence numbers. (In English)

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The K_p-independence number $\alpha_p(G)$ of a graph G is the maximum order of an induced subgraph of G that contains no K_p. In this paper, the authors study the following problem: For integers r, p, $m > 0$ and graphs $L_1, \ldots, L_r$, what is the maximum number of edges in a graph G of order n such that $\alpha_p(G) \leq m$ and there is an edge-colouring of G with r colours such that the jth colour class contains no copy of L_j, for $j = 1, \ldots, r$ (this maximum number is denoted by $\mathrm{RT}_p(n, L_1, \ldots, L_r, m)$)? They obtain several results for graphs G of order n with small K_p-independence number $\alpha_p(G)$. Some structure theorems are given for the case $\alpha_p(G_n) = o(n)$, showing that there are graphs with fairly simple structure that are within $o(n^2)$ of the extremal size; the structure is described in terms of the edge densities between certain sets of vertices. They also study the problem of determining the asymptotic value of

$$\theta_p(K_q) = \lim_{n\to\infty} \frac{\mathrm{RT}_p(n, K_q, o(n))}{\binom{n}{2}}$$

for $p < q$. They list several open problems and make some conjectures in the paper.

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Classification:
05C35 Extremal problems (graph theory)
05C55 Generalized Ramsey theory
05C15 Chromatic theory of graphs and maps
05C50 Graphs and matrices

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