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On partitions of lines and space. (In English)
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The main theorem of this paper is:
Suppose θ is 0 or a limit ordinal, $s \in \omega$, and $\theta + s \geq 1$. Then $2^\omega \leq \omega_{\theta+s}$ is equivalent to the statement: For every $n \geq 2$ and partition $L_1 \cup L_2 \cup \cdots \cup L_p$ of the lines of $\mathbb{R}^n$, there is a partition $\mathbb{R}^n = S_1 \cup S_2 \cup \cdots \cup S_p$ such that each line in L_i meets S_i in a set of size $\leq \omega_{\theta+s-p+1}$. (In the case $\theta = 0$ and $s-p+1 < 0$, then each line in L_i meets S_i in a finite set.)
This yields as a corollary a classical result of Sierpiński that $2^\omega = \omega_1$ is equivalent to the statement that there exists a partition $\mathbb{R}^3 = S_1 \cup S_2 \cup S_3$ such that for each i for every line l parallel to the i-axis, $l \cap S_i$ is finite. It and variations prove generalizations of Sierpiński which are due to Kuratowski, Sikorski, Erdős, Davies, Bagemihl, Simms, Bergman, Hrushovski, Galvin, and Gruenhage.
In the case of partitions into infinitely many pieces, their main result is: If the lines L in $\mathbb{R}^n$ ($n \geq 2$) are partitioned into ω disjoint pieces $L = \bigcup_{i<\omega} L_i$, then there is a partition $\mathbb{R}^n = \bigcup_{i<\omega} S_i$ such that for each i every line in L_i meets S_i in a finite set.
A related question about partitions on ω_1 is considered, and using the Axiom of Determinateness (AD), the following result is obtained: (ZF + AD + DC) For every partition $[\omega_1]^2 = \bigcup_{n<\omega} Q_n$ there is a partition $\omega_1 = \bigcup_{n<\omega} A_n$ such that $[A_k]^2$ meets only finitely many Q_n for each $k < \omega$.
This contrasts sharply with a result of Todorcevic that in ZFC there exists a partition $[\omega_1]^2 = \bigcup_{n<\omega} Q_n$ such that for every uncountable $A \subset \omega_1$, $[A]^2$ meets every Q_n.

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Classification:
04A20 Combinatorial set theory
03E60 Axiom of determinacy, etc.
04A30 Continuum hypothesis and generalizations

Keywords:
value of continuum; partitions of lines in Euclidean space; axiom of determinateness