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How many edges should be deleted to make a triangle-free graph bipartite? (In English)

Halász, G. (ed.) et al., Sets, graphs and numbers. A birthday salute to Vera T. Sós and András Hajnal. Amsterdam: North-Holland Publishing Company, Colloq. Math. Soc. János Bolyai. 60, 239-263 (1992). [ISBN 0-444- 98681-2/hbk]

We shall prove that if L is an arbitrary 3-chromatic graph and G_n is a simple graph on n vertices not containing L, and having at least $\frac{n^2}{5} - o(n^2)$ edges, then it can be made bipartite by throwing away at most $\frac{n^2}{25} - o(n^2)$ edges. This was known for $L = K_3$.

Let us call a graph pentagonlike if we can colour its 5 classes so that the vertices coloured by i are joined only to vertices coloured by $i \pm 1 \pmod 5$.

In addition to the above assertions, we shall prove that under the above conditions, there is a "pentagonlike graph" H_n with $e(H_n) = e(G_n)$ for which we have to delete more edges than in case of G_n to make it bipartite. We shall also prove a related stability theorem, according to which, if $D(G_n)$ denotes the minimum number of edges to be deleted to make G_n bipartite, then either $D(G_n) \leq D(H_n) - cn^2$; i.e. G_n is significantly better than H_n—though they both may be far from any bipartite graph—or the structure of G_n is very near to that of a pentagonlike graph.

Classification:
05C35 Extremal problems (graph theory)
05C15 Chromatic theory of graphs and maps

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stability theorem