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Erdős, Paul; Sárközy, A.
On sets of coprime integers in intervals. (In English)
Hardy-Ramanujan J. 16, 1-20 (1993).

For any subset $\mathcal{A}$ of $\mathbb{N}$, let $\Phi_k(\mathcal{A})$ denote the number of k-tuples $(a_1, \dots, a_k)$ with $a_i \in \mathcal{A}$, $a_1 < a_2 < \cdots < a_k$ and $(a_i, a_j) = 1$ for $1 \le i < j \le k$ and let Γ_k denote the family of those $\mathcal{A}$ with $\Phi_k(\mathcal{A}) = 0$. Further, define $g_k(m,n)$ to be the maximal cardinality of a set $\mathcal{A} \in \Gamma_k$ and lying in $[m, m+n-1]$ and write $F_k(n) = g_k(1,n)$ and $G_k(n) = \max_{m\in\mathbb{N}} g_k(m,n)$. Define also the functions $\psi_k(m,n)$ and $\Psi_k(n)$ by $\psi_k(m,n) = |\{u \in \mathbb{N} : u \in [m, n+n-1]$, u divisible by at least one of the first k primes$\}|$ and $\Psi_k(n) = \psi_k(1,n)$. Finally, let $h_{(k,l)}(m,n)$ denote the maximal cardinality of a set $\mathcal{A} \in \Gamma_l$ and lying in $[m, n+n-1]$ with the property that each $a \in \mathcal{A}$ is not divisible by any of the first k primes.

The first author has conjectured that $F_k(n) = \Psi_{k-1}(n)$ for each k and this has been confirmed for $k \le 4$. In this paper, the authors study the case of general k and prove seven theorems concerning connections between the various functions mentioned above.

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