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Erdős, Paul; Galvin, Fred
Some Ramsey-type theorems. (In English)
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For any set A let $[A]^r$ denote the collection of r-element subsets of A. By a k-coloring of the r-subsets of A we mean a function $f : [A]^r \to \{1,\dots,k\}$. A set $X \subset A$ is said to be f-homogeneous if f is a constant on $[X]^r$. The partition symbol $a \to (x)^r_k$ denotes the assertion: given a set A with $|A| = a$ and a coloring $f : [A]^r \to \{1,\dots,k\}$, there is an f- homogeneous set $X \subset A$ with $|X| \le x$.
The main result of this paper is
Theorem 2.1. Let r and k be positive integers, and let the function $\varphi : \mathbb{N} \to \mathbb{R}$ be such that $n \to (\varphi(n))^r_{k+1}$ holds for all sufficienty large n. Given any coloring $f : [\mathbb{N}]^r \to \{1,\dots,k\}$, there is a set $A \subset \mathbb{N}$ such that: (1) $|\{f(X) : X \in [A]^r\}| \le 2^{r-1}$; (2) $|A \cap \{1,\dots,n\}| \ge \varphi(n)$ for infinitely many n.

J.E.Graver (Syracuse)

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05D10 Ramsey theory

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