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Erdős, Paul; Makai, Endre; Pach, János; Spencer, Joel

Gaps in difference sets, and the graph of nearly equal distances. (In English)

Applied geometry and discrete mathematics, Festschr. 65th Birthday Victor Klee, DIMACS, Ser. Discret. Math. Theor. Comput. Sci. 4, 265-273 (1991).

[For the entire collection see Zbl 726.00015.]

Given n real numbers mutually differing by at least 1, let $1 \leq d_1 \leq \cdots \leq d_m$, $m = \binom{n}{2}$, denote the increasing sequence of differences between them. The authors prove the asymptotically sharp bound

$$\sum_{j=1}^{m-1} (d_{j+1} - d_j)^2 > c \cdot \log n,$$

and give sharp lower bounds for $\sum_{j=1}^{m-1} \phi(d_{j+1} - d_j)$ for large class of monotone increasing convex functions ϕ. They also show that for n points in the plane with minimal distance 1 and any real number t at most $\lceil n^2/4 \rceil$ may have distances between t and $t+1$, and give an analogous result in terms of the diameter of the point set.

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52A37 Other problems of combinatorial convexity
05B10 Difference sets
52C10 Erdoes problems and related topics of discrete geometry

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difference set; distance set