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Variations on the theme of repeated distances. (In English)

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Given a set $X = \{x_1, ...x_n\}$ of n points in $\mathbb{R}^d$, let $f(X)$ denote the number of pairs $\{x_i, x_j\}$ whose Euclidean distances $||x_i - x_j|| = 1$. Let

$$f_d(n) = \max_{X \subset \mathbb{R}^d, \quad |X| \leq n} f(X).$$

An asymptotically sharp estimate for the error term in this maximum is given for $d \geq 4$. Tight upper bounds are also determined for the total number of occurrences of what are called "favourite" distances from n points in $\mathbb{R}^d$, $d \geq 4$. Some related results are also proved for distances determined by n disjoint compact convex sets in $\mathbb{R}^2$.

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Classification:
52C10 Erdoes problems and related topics of discrete geometry
52A10 Convex sets in 2 dimensions (including convex curves)
52A20 Convex sets in n dimensions

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repeated distances; points in $\mathbb{R}^d$; convex sets