
Zbl 699.10069**Brown, T.C.; Erdős, Paul; Freedman, A.R.***Quasi-progressions and descending waves.* (In English)**J. Comb. Theory, Ser. A 53, No.1, 81-95 (1990). [0097-3165]**

Several generalizations of the property of having arbitrarily long arithmetic progression (abbreviated AP) and their mutual dependence is investigated. These properties are: to have arbitrarily large “quasi progressions” (QP), arbitrarily large “combinatorial progressions” (CP), arbitrarily large “descending waves” (DW) or arbitrarily large “cubes” (C). Most of the corresponding definitions are too complicated to be reproduced here, but e.g. cubes are sets of the form $(a + \epsilon_1 y_1 + \dots + \epsilon_m y_m; \epsilon_j = 0 \text{ or } 1, 1 \leq j \leq m)$. The authors prove that $AP \Rightarrow QP \Rightarrow CP \Rightarrow C \Rightarrow DW$, whereas none of these implications is reversible. They also show that the famous conjecture that every set of positive integers with infinite reciprocal sum has property AP is equivalent to the statement that every set with infinite reciprocal sum has property QP. For descending waves an analogon for the van der Waerden result is proved, etc.

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11B25 Arithmetic progressions

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Szemerédi's proof, Erdős conjecture; arbitrarily long arithmetic progression; quasi progressions; combinatorial progressions; descending waves; cubes