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Erdős, Paul; Nicolas, J.L.
On functions connected with prime divisors of an integer. (In English)
Number theory and applications, Proc. NATO ASI, Banff/Can. 1988, NATO ASI Ser., Ser. C 265, 381-391 (1989).
[For the entire collection see Zbl 676.00005.]

For an arithmetical function g an integer n is called a g-champion if $g(m) < g(n)$ for all $m < n$. Let $n = q_1^{\alpha_1} q_2^{\alpha_2} ... q_k^{\alpha_k}$, where $q_1 < q_2 < ... < q_k$ are primes. In this paper four arithmetical functions are considered, namely

$$f(n) = \sum_{i=1}^{k-1} \frac{q_i}{q_{i+1}}; \quad F(n) = \sum_{i=1}^{k-1} (1 - \frac{q_i}{q_{i+1}});$$

$$h(n) = \sum_{i=1}^{k-1} \frac{1}{q_{i+1} - q_i}; \quad \hat{h}(n) = \sum_{1 \leq i < j \leq k} \frac{1}{q_j - q_i}.$$

Previously, the authors had shown that $n(x) = \prod_{p \leq x} p$, $p =$ prime, is an f-champion for all large x, but not an F-champion for large x [Théorie des nombres, C. R. Conf. Int., Quebec/Can. 1987, 169- 200 (1989; Zbl 683.10035)]. Here the authors first show that $n(x)$ is not an h-champion, by using a deep result due to Maier on chains of long gaps between primes. Next, they show that $n(x)$ is not an $\hat{h}-$champion by assuming two strong conjectures one of which is that in short intervals of the form $(x, x + x^\epsilon)$ one can find the expected number of primes. Their second assumption is that there exist 4-tuples of primes $p, p+2, p+6, p+8$ in short intervals of the form $x, x + x^{1/100}$.

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11N05 Distribution of primes

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champion numbers; primes in short intervals; gaps between primes