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Erdős, Paul; Vértesi, P.
On certain saturation problems. (In English)
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In the present paper, the author studies certain saturation problems on the interpolatory linear operators

$$(L_n f)(x) = \sum_{0\le k\le n} f(k/n)|x-k/n|^{-r} / \sum_{0\le k\le n} |x-k/n/|^{-r}; \quad 0\le x\le 1, \quad n\ge 1,$$

where $r > 2$ is a fixed real number, $f \in C[0,1]$, and gives two theorems on it. His main result is as follows: "Theorem: Let $0 < x_0 < 1$ be a fixed irrational number, $\{y_r\}_{r-1}^{\infty}$ be an arbitrary sequence with $y_r \neq x_0$, $r = 1,2,...$, $\lim_{r\to\infty} y_r = x_0$. Further let $0 < p{*} \le 1/3$ (real), $p,q > 0$, $(p,q) = 1$ (integers), $0 \le \gamma < p$, $0 \le \delta < q$ (reals) be fixed numbers. Then there exist a sequence $\{x_k\} \subset \{y_r\}$ and positive integers $\{\ell_k\}_{k=1}^{\infty}$ and $\{n_k\}_{k=1}^{\infty}$ with $1 < n_1 < n_2 < ....$ i.e. $\lim_{k\to\infty} n_k = \infty$ such that relations

$$|x_0 - (p\ell_k + \gamma)/(qn_k + \delta)| = o(1/n_k), \quad k = 1,2,...,$$

and

$$p*/2n_k \le |x_k - (p\ell_k + \gamma)/(qn_k + \delta)| \le (2p+2)p/n_k, \quad k = 1,2,...,$$

hold true.

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Classification:
41A40 Saturation
41A05 Interpolation
41A36 Approximation by positive operators

Keywords:
diophantine equation; modulus of continuity; saturation problems; interpolatory linear operators