## Zbl 645.10045

Erdős, Paul; Nathanson, Melvyn B.

Partitions of bases into disjoint unions of bases. (In English)

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Two Ramsay-like combinatorial results on partitions are proved using probabilistic methods and the Borel-Cantelli lemma. The authors deduce that if A is an asymptotic basis of order h and if every large integer has sufficiently many representations as a sum of h elements of A, then A is a union of a finite or infinite number of pairwise disjoint asymptotic bases of order h.

Waring's problem is extended to showing that for each  $k \geq 2$  and for all  $s > s_0(k)$ , the set  $A = \langle n^k : n = 1, 2, ... \rangle$  has a partition  $A = \bigcup_{j=1}^{\infty} A_j$  such that each  $A_j$  is an asymptotic basic of order s. In the other direction, they show that the squares cannot be partitioned into disjoint sets which are asymptotic bases of order 4; for numbers not divisible by 4 there is a positive result. Some open problems are also included. For another combinatorial result which also has applications to additive number theory, see P. Erdős and R. Rado [Intersection theorems for system of sets, J. Lond. Math. Soc. 35, 85-90 (1960; Zbl 103.27901)] and the reviewer [Homogeneous additive congruences, Philos. Trans. R. Soc. Lond., Ser. A 261, 163-210 (1967; Zbl 139.27102)].

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## Classification:

11B13 Additive bases

11B75 Combinatorial number theory

11P05 Waring's problem and variants

05C55 Generalized Ramsey theory

05A05 Combinatorial choice problems

11P81 Elementary theory of partitions

## Keywords:

asymptotic basis of order h; Waring's problem; partition