Zbl 642.10021

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Sur le nombre d'invariants fondamentaux des formes binaires.

On the number of fundamental invariants of binary forms. (In French)

C. R. Acad. Sci., Paris, Sér. I 305, 319-322 (1987). [0764-4442]

For an arbitrary positive integer d, let ω_d denote the number of fundamental invariants for binary forms of degree d. For odd $d \geq 3$, *V. G. Kac* [Lect. Notes Math. 996, 74-108 (1983; Zbl 534.14004)] minorized ω_d in the following way. For an arbitrary positive integer n, consider the congruence $x_1 + 2x_2 + ... + (n-1)x_{n-1} \equiv 0(\mod n)$ where $x_1, ..., x_{n-1}$ are nonnegative integers, not all 0. A solution is said to be indecomposable if it is not the sum of two solutions. Denoting the number of solutions indecomposable by $F(n)$, *V.G.Kac* [ibid.] proved that

$$(1) \quad \omega_d \geq F(d-2) \quad for \quad odd \quad d \geq 3.$$

Let $p(n)$ denote the number of unrestricted partitions of n (and ϕ Euler's totient function). (1) immediately implies that

$$(2) \quad \omega_d \geq p(d-2) + \phi(d-2) - 2 \text{ for odd } d \geq 3.$$

In the paper under review, the authors improve the lower bounds of $F(n)$ and obtain that

$$(3) \quad \omega_d \gg p(d) \cdot d^{1/2} (\log d \log\log d)^{-1} \text{ for odd } d.$$

As to the even d's, using two results of type (1), proved by *V. L. Popov* [Izv. Akad. Nauk SSSR, Ser. Mat. 47, No.3, 544-622 (1983; Zbl 573.14003); J. Reine Angew. Math. 341, 157-173 (1983; Zbl 525.14007)], the authors obtain that

$$\omega_d \gg p(d/4) \cdot d^{1/2} (\log d \log\log d)^{-1} \text{ for } d \equiv 0 (\mathrm{mod} 4)$$

and with $\epsilon > 0$

$$\omega_d \gg_\epsilon^p (d/2) \cdot \exp\{(\pi 3^- - \epsilon) d^{1/2} \log\log d (\log d)^{-1}\} \text{ for } d \equiv 2 (\mathrm{mod} 4).$$

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Classification:
11E76 Forms of degree higher than two
15A72 Vector and tensor algebra
11P81 Elementary theory of partitions

Keywords:
binary forms of higher degree; fundamental invariants; partitions