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Erdős, Paul; Murty, M.Ram; Murty, V.Kumar (Ram Murty, M.; Kumar Murty, V.)

On the enumeration of finite groups. (In English)

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Let $G(n)$ denote the number of non-isomorphic groups of order n. Using methods of analytic number theory and an explicit algebraic equation of Hölder, the authors derive interesting asymptotic information about $G(n)$ when n is square-free. The main results are:

$$(i) \quad G(n) = \Omega(n^{1-\epsilon}) \text{ for every } \epsilon > 0 \text{ when } n \text{ is square-free,}$$

$$(ii) \quad \log G(n) = (1 + o(1)) \ \log\log n \sum_{p|n} (\log p)/(p-1)$$

for almost all square-free n. In addition, they derive asymptotic estimates for $F_k(x)$ where $F_k(x) = card\{n \leq x : G(n) = k\}$:

$$F_k(x) = (c(a) + o(1))x/(\log\log\log x)^{a+1} \text{ for } k = 2^a,$$

$$F_k(x) = O(x/(\log\log x)^{1-\epsilon}) \text{ for } k \neq 2^a,$$

where $c(a)$ is an appropriate constant.

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Classification:
11N45 Asymptotic results on counting functions for other structures
20D60 Arithmetic and combinatorial problems on finite groups

Keywords:
number of non-isomorphic groups; asymptotic estimates