Zbl 593.10036

Erdős, Paul

On two unconventional number-theoretic functions and on some related problems. (In English)

Calcutta Math. Soc. Diamond-Cum-Platinum Jubilee Commem. Vol. (1908- 1983), Pt. 1, 113-121 (1984).

[For the entire collection see Zbl 584.00012.]

The author proves a number of results and formulates conjectures about two number-theoretic functions related to the distribution of the prime divisors of an integer. One of the two functions is defined as

$$f(n) = \sum_{p|n, p^{\alpha} \le n < p^{\alpha+1}} p^{\alpha}.$$

Among other things, the author shows that $m(x) = \max_{n \le x} f(n)$ satisfies

$$m(x) \le (1 + o(1))x \log x / \log \log x$$
 as $x \to \infty$,

and conjectures that in this bound one has asymptotic equality. He further states that the logarithmic density of the set of integers n satisfying $f(n) \leq cn$ exists for any c and is a continuous function of c.

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Classification:

11N37 Asymptotic results on arithmetic functions

11K65 Arithmetic functions (probabilistic number theory)

Keywords:

arithmetic functions; conjectures; prime divisors; logarithmic density