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Polychromatic Euclidean-Ramsey theorems. (In English)

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The Euclidean Ramsey Property (ERP) for a set S in Euclidean space E^n is that for every integer $r > 0$ there exists a sufficiently large integer N such that for all $m \geq N$ and every r-coloring of E^m there exists a monochromatic set S' in E^m congruent to S. In earlier papers the first author proved that a necessary condition for ERP is that S be a finite subset of a sphere and, more generally, that if S has a k-chromatic congruent copy in all r-colorings of sufficiently high dimensional Euclidean spaces (called k-ERP), then S must be embeddable in k concentric spheres. The authors investigate sets which are exactly k-ERP (possess the k-ERP property but not $(k-1)$-ERP). The key to the construction of such sets is the existence of a highly transitive group of isometries (i.e., either the alternating or the symmetric group) acting on a family of subsets of a large set, and the concept of simplicial ERP introduced in the paper. The result from which essentially all other results and examples follow is: Let $0 \leq i_1 \leq i_2 \leq ... \leq i_k \leq n-1$ and let P_i denote the set of centroids of the i-sub-simplices of a regular simplex S_n. Then the set $S = P_{i_1} \cup P_{i_2} \cup ... \cup P_{i_k}$ has the exact k-ERP. An example of a set having the 3-ERP but not the 2-ERP is that consisting of the vertices of a non-obtuse, non-equilateral isosceles triangle and the trisecting points of its sides. A number of unsolved problems and conjectures are stated.

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