
Zbl 539.10038

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Lagrange's theorem and thin subsequences of squares. (In English)

Contributions to probability, Collect. Pap. dedic. E. Lukacs, 3-9 (1981).

[For the entire collection see Zbl 527.00019.]

Lagrange showed that every natural number is the sum of at most four squares of integers. In this paper the question is raised whether there are "thin" subsequences A of squares such that every natural number can still be expressed as the sum of at most four squares in A. An answer is given by the following theorem: For every $\epsilon > 0$ there exist a subsequence A_ϵ of squares and a constant $c = c(\epsilon) > 0$ such that every natural number is the sum of at most four squares in A_ϵ and $(*)$ $A_\epsilon(x) := |A_\epsilon \cap [1, x]| \leq cx^{3/8+\epsilon}$ [c.f. *M. B. Nathanson*, Lect. Notes Math. 899, 301-310 (1981; Zbl 476.10042)]. The authors conjecture that the exponent in (*) can be reduced to $1/4 + \epsilon$ (meanwhile proved by the reviewer; to appear).

Two other theorems state similar properties on the representation of integers $n \neq 4^k(8m + 7)$ as the sum of three squares and on the representation of even integers as the sum of two products of at most two odd primes. Here the exponents in the inequalities corresponding to (*) are - apart from the ϵ- best possible ($1/3 + \epsilon$ and $1/2 + \epsilon$ respectively).

All these results are based on a general lemma, which is proved by probabilistic methods developed by the first author and *A. Rényi* [Acta Arith. 6, 83-110 (1960; Zbl 091.04401)].

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Classification:
11P05 Waring's problem and variants
11B13 Additive bases
11P32 Additive questions involving primes
11K99 Probabilistic theory

Keywords:
sums of squares; additive bases; probabilistic methods; representation of integers; sum of at most four squares; sum of three squares; representation of even integers; sum of two products of at most two odd primes