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Erdős, Paul; Mullin, R.C.; Sos, V.T.; Stinson, D.R.
Finite linear spaces and projective planes. (In English)
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A nondegenerate finite linear space (NLS) is defined such that every two points occur in a unique line, every line contains ≥ 2 points, and no line contains all but one of the points. The number of lines and of points are, respectively, called b and v. *N.G.de Bruijn* and *P.Erdős* [Indag. Math. 10, 1277-1279 (1948; Zbl 032.24405)] have shown that $b \geq v$ holds in an NLS, with equality iff the NLS is a projective plane. In the paper under review, the authors show that in an NLS with $v \geq 5$ one has $b \geq B(v)$ where

$$B(v) = \begin{cases} n^2+n+1 & \text{if } n^2+2 \leq v \leq n^2+n+1 \\ n^2+n & \text{if } n^2-n+3 \leq v \leq n^2+1 \\ n^2+n-1 & \text{if } v = n^2-n+2, \end{cases}$$

with equality if n is the order of a projective plane. Moreover, minimal NLS's (that is, if no NLS on v points has fewer lines) and their embeddability in projective planes are studied. The paper concludes with a few open problems.

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Classification:
51A45 Incidence structures imbeddable into projective geometries
05B25 Finite geometries (combinatorics)
51E15 Affine and projective planes

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