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Erdős, Paul; Silverman, R.; Stein, A.

Intersection properties of families containing sets of nearly the same size. (In English)

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This paper addresses the question: one wants to choose a set S of points in a projective plane of order n such that every line contains at least one point of S , and such that no line has more than x members of S . How small can x be: It is shown that x can be chosen of the order $c \log n$. One can argue as follows: if one chooses every point with probability P at random, the distribution of the number of points on any line obeys a binomial distribution and can be explicitly evaluated. If p is chosen such that the probability of obtaining either 0 or x or more points on a line is less than $\left(\frac{1}{n^2+n+1}\right)$ or $\left(\frac{1}{n^2+n+1}\right)^{-1}$, then there is a finite probability that no line in the plane falls to intersect the chosen points in at least one, but x or fewer, points. This paper contains detailed computations showing that this occurs for $x = c \log n$ and appropriate c . It also contains discussion of a less efficient constructive example.

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Classification:

05A05 Combinatorial choice problems

05B25 Finite geometries (combinatorics)

60C05 Combinatorial probability

Keywords:

families of finite sets; projective plane; binomial distribution