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Some asymptotic formulas on generalized divisor functions. I. (In English)

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[This article was published in the book announced in Zbl 512.00007.]
For a given sequence $A = \{a_1 y a_2 < \dots\}$ of positive integers, define the counting function $N_A(x) = \sum_{a\in A, A\le x} l$, the logarithmic counting function $f_A(x) = \sum_{a\in A, A\le x} a^{-l}$, and the generalized divisor function $\tau_A(n) = \sum_{a|n, a\in A} l$; denote its maximum by $D_A(x) = \max_{1\le n\le x} \tau_A(n)$. The authors' aim is to look for "large" values of $D_A(x)$ and to check the truth of the "natural" conjecture

$$(*) \qquad \lim_{x\to\infty} \frac{D_A(x)}{f_A(x)} = \infty, \text{ if } N_A(x) \to \infty.$$

However, the authors disprove this conjecture: There exists an infinite sequence A such that $\limsup_{x\to\infty} f_A(x)/\log\log x > 0$, but $\sup D_A(x)/f_A(x)$ is finite. Then the authors prove theorems giving large values for $D_A(x)$, for example: If $\lim_{x\to\infty} f_A(x) = \infty$, then

$$\limsup_{x\to\infty} D_A(x)/f_A(x) = \infty.$$

This theorem shows that a weakened version of conjecture (*) is true. {Parts II-IV of this series are published in J. Number Theory 15, 115-136 (1982; Zbl 488.10043), Acta Arith. 41, 395-411 (1982; Zbl 492.10037), Stud. Sci. Math. Hung. 15, 467-479 (1980; Zbl 512.10037)}.

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