**Erdős, Paul; Sós, Vera T.**
*On Ramsey - Turan type theorems for hypergraphs.* (In English)
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Let $H^r$ be an r-uniform hypergraph and $f(n; H^r)$ be the minimal integer so that any $r$-uniform hypergraph on $n$ vertices and more than $f(n; H^r)$ edges contains a subgraph isomorphic to $H^r$. The extimation of $f(n; H^r)$ is the fundamental problem of extremal graph theory. The paper deals with extremal theory for hypergraphs. The main result is that the situation is quite different for hypergraphs. To be more precisely, let $f(n; H^r, \ell)$ be the smallest integer for which every graph of $n$ vertices and more than $f(n; H^r, \ell)$ edges either contains a subgraph isomorphic to $H^r$ or it contains an independent set of size $\ell$. Let, moreover, $E = \{h_1, \ldots, h_m\}$ be the edge-set of $H^r$ such that, for every $i$, $1 \leq i \leq m$, there exists a $j \neq i$ with $|h_i \cap h_j| \geq 2$. If $\lim_{n \to \infty} \binom{n}{r}^{-1} f(n; H^r) = c(H^r)$ and $\lim_{\varepsilon \to 0} \lim_{n \to \infty} \binom{n}{r}^{-1} f(n; H^r, \varepsilon n) = c^*(H)$ then $c^*(H^r) = c(H^r)$. There are also given conditions ensuring $c^*(H^r) = 0$. The authors state also 10 open problems; a few of them concern graphs of uniform edge density.

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05C65 Hypergraphs
05C35 Extremal problems (graph theory)
05C55 Generalized Ramsey theory

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r-uniform hypergraph; edge density; spanned subgraph