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On sums involving reciprocals of certain arithmetical functions. (In English)

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The paper investigates certain sums involving reciprocals of the functions $\omega(n)$, $\Omega(n)$ and $P_i(n)$. Here $\omega(n)$ denotes the number of distinct prime factors of n, $\Omega(n)$ denotes the number of all prime factors of n, and if $\omega(n) \geq i$ then $P_i(n)$ is the i-th largest prime factor of n, i.e.

$$P(n) = P_1(n) > P_2(n) > \cdots > P_i(n) > \cdots > P_{\omega(n)}(n)$$

are the distinct prime factors of n. It is shown that the sums $\sum_{2\leq n\leq x} n^{-1/\omega(n)}$ and $\sum_{2\leq n\leq x} n^{-1/\Omega(n)}$ are of the order $x\exp(-A(\log x \cdot \log\log x)^{1/2})$ and $x\exp(-B(\log x)^{1/2})$ respectively, where the upper and lower bounds for $A, B > 0$ are explicitly calculable. Next it is shown that

$$(\log x/\log\log x)^{1/2} \sum_{2\leq n\leq x} 1/P(n) \ll \sum_{2\leq n\leq x} \Omega(n)/P(n) \ll$$

$$\ll (\log x \log\log x)^{1/2} \sum_{2\leq n\leq x} 1/P(n), \tag{1}$$

and in a joint forthcoming paper with C. Pomerance (1) will be further sharpened. Finally it is shown that with some $B_i > 0$ and $i \geq 2$ fixed

$$\sum\nolimits'_{n\leq x} 1/P_i(n) = (B_i + 0(1))x(\log\log x)^{i-2}/\log x,$$

where the dash' denotes summation over those n for which $\omega(n) \geq i$.

Classification:

11N05 Distribution of primes

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number of distinct prime factors; number of prime factors; largest prime factor