Zbl 491.41001
Erdős, Paul; Vertesi, P.
On the almost everywhere divergence of Lagrange interpolation. (In English)
Approximation and function spaces, Proc. int. Conf., Gdansk 1979, 270- 278 (1981).

[For the entire collection see Zbl 471.00018.]
In a previously published paper, *P.Erdős* [Acta Math. Acad. Sci. Hung. 9, 381-388 (1958; Zbl 083.29001)] stated without proof that if $(x_{kn})_{1\leq k\leq n,1\leq n}$ denotes a triangular matrix of knots in the compact internal $I=[-1,+1]$ ordered such that $x_{n,n}<x_{n-1,n}<\cdots<x_{1,n}$ $(n\geq 1)$ holds then there exists a continuous function $f:I\to\mathbb{R}$ such that the sequence $(L_nf)_{n\geq 1}$ of Lagrange interpolation polynomials $L_n=\sum_{1\leq k\leq n}f(x_{kn})\ell_{kn}$ diverges almost everywhere in I. In fact $\lim.\sup n\to\infty|L_nf(x)|=\infty$ for almost all $x\in I$. The authors give a brief account of the preliminary results in this direction (G. Faber, S. Bernstein, G. Grünwald, J. Marcinkiewicz, A. A. Privalov, P. Turán, P. Erdős) and point out a sketch of the proof. The detailed proof is rather long and quite complicated, although it uses only elementary techniques. One of its important ingredients is the following result: Lemma. Let $A>0$ be an arbitrary fixed number and consider an arbitrary integer $m\geq m_0(A)$. For any integer $n\geq n_0(m)$ exists a set $H_n\subset I$ for which $\text{meas}(H_n)\leq 1/\ln\ln m$ and

$$\sum_{\overline{1\leq k\leq n x_k\notin I_{j(x),n}}}|\ell_{kn}(x)|\geq(\ln m)^{1/3}geq2A\ (n\geq n_0(m))$$

where $I_{j(x),m}$ denotes that interval of the equivalent partition of I in m subintervals which contains the point $x\in I|H_n$. — Details of the proof (about 300 pages) will be published in a paper to appear in Acta Math. Acad. Sci. Hung.

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almost everywhere divergence; triangular matrix of knots; Lagrange interpolation polynomials